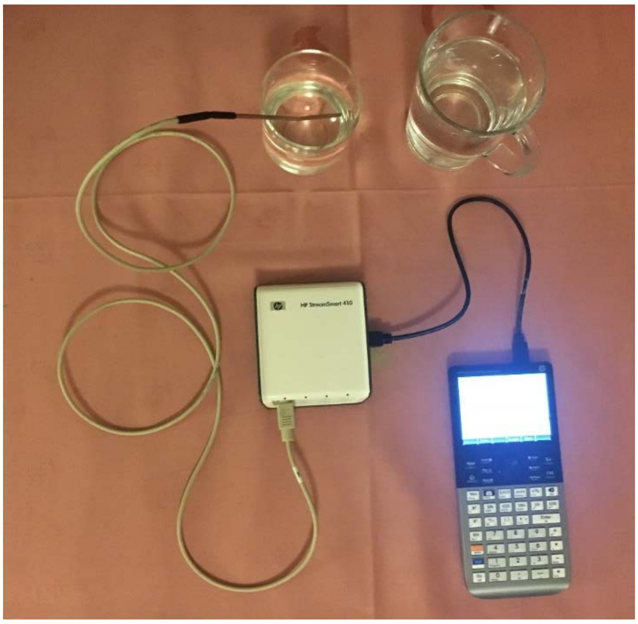
HP Prime Application Note Physics:

08. Newton's Law of Warming and Cooling

In this experiment, we measure warming and cooling processes, numerically modelling the experimental data according to Newton's law of warming and cooling. For the experiment, we use Hewlett-Packard's Mobile Calculating Laboratory (MCL), which includes the interfacing module HP StreamSmart 410 and the Fourier temperature sensor DT029, as shown in the picture below.

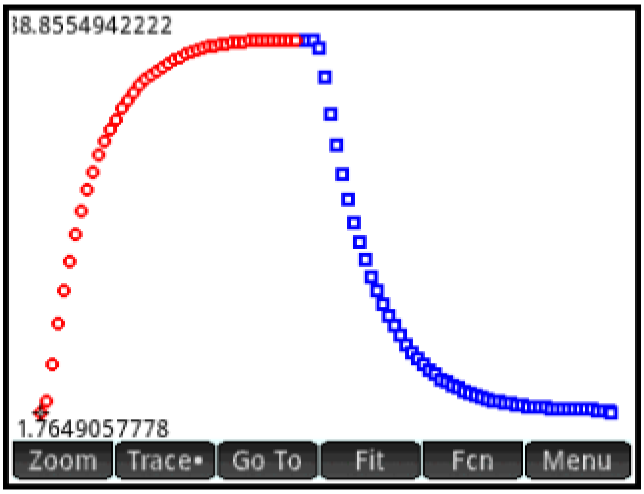
The accuracy of the Fourier temperature sensor DT029 amounts to ±2 % with a resolution of 0.03 °C over the entire range from -25 to 110 °C. As the HP Prime started logging the time and the temperature, the sensor was taken out of the glass with water at room temperature and put into the cup with hot water. After the recorded temperature reached its maximum, the temperature sensor was immediately put back into the glass with water at room temperature. The entire warming and subsequent cooling experiment lasted less than 70 seconds.



The original data are listed in the Appendix. The temperature (*y* axis) is plotted versus the time (*x* axis) in the graphs on the HP Prime below:

In the graph, the red circles and blue squares indicate the warming and cooling trajectories respectively.

Newton's law states that the rate of temperature change is proportional to the temperature difference between the body and the environment, which boils down to the following differential equation: $\frac{dT}{dt} = k \cdot T$. We will discuss the two solutions of the differential equation.

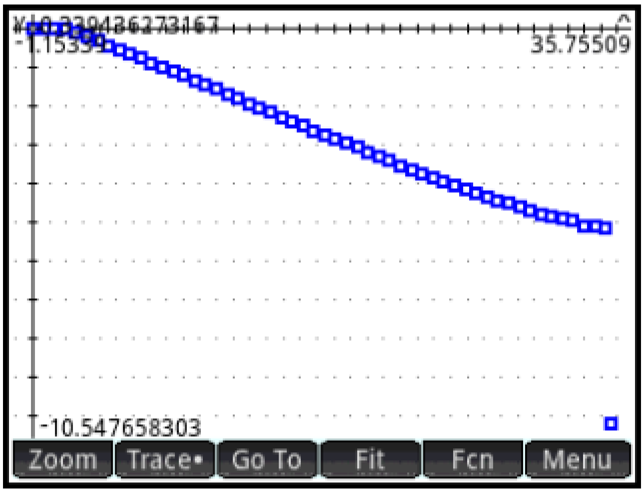


For cooling ($k < 0$), the following relation holds for temperature $T(t)$ at time *t*:

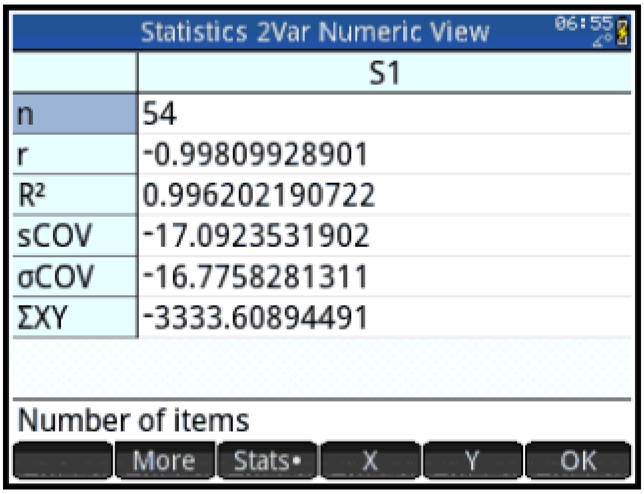
$$T(t) = T_{\text{room}} + (T_{\text{start}} - T_{\text{room}}) \cdot e^{k \cdot t} \leftrightarrow$$

$$\ln\left[\frac{T(t) - T_{\text{room}}}{T_{\text{start}} - T_{\text{room}}}\right] = k \cdot t$$

In our cooling experiment, $T_{\text{start}} = 84.14$ °C and $T_{\text{room}} = 16.43$ °C. We count the time of the cooling process, which starts at point no. 45 (*t* = 33.9 s), and set it to zero. We subsequently explore the relationship between $y \equiv \ln\left[\frac{T(t) - T_{\text{room}}}{T_{\text{start}} - T_{\text{room}}}\right]$ versus *t*, which is a line through the origin with a negative slope *k*. Using the Spreadsheet app to calculate *y*, copy these values into the Statistics 2Var app, and plot *y* versus *t* (*x* axis):



Using a least-squares method, the slope of a line through the origin is: $k = \frac{\sum x \cdot y}{\sum x^2}$.
Ignoring the last point, the values of $\sum x \cdot y$ and $\sum x^2$ are listed in the Stats menu of the Statistics 2Var app:



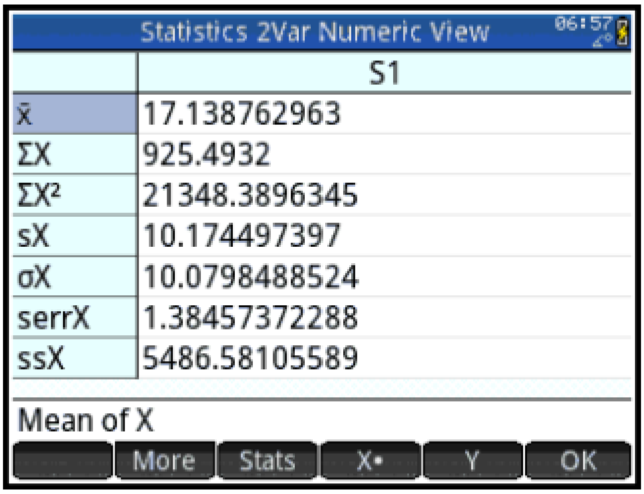
Statistics 2Var Numeric View

	S1
n	54
r	-0.99809928901
R²	0.996202190722
sCOV	-17.0923531902
σCOV	-16.7758281311
ΣXY	-3333.60894491

Number of items

More | Stats• | X | Y | OK

So we now have determined $k = \frac{\sum x \cdot y}{\sum x^2} = \frac{-3333.61}{21348.39} = -0.156$ and thus numerically modelled the cooling process: $\boldsymbol{T(t)} = \mathbf{16.43 + 67.71 \cdot e^{-0.156 \cdot (t - 33.90)}}$.

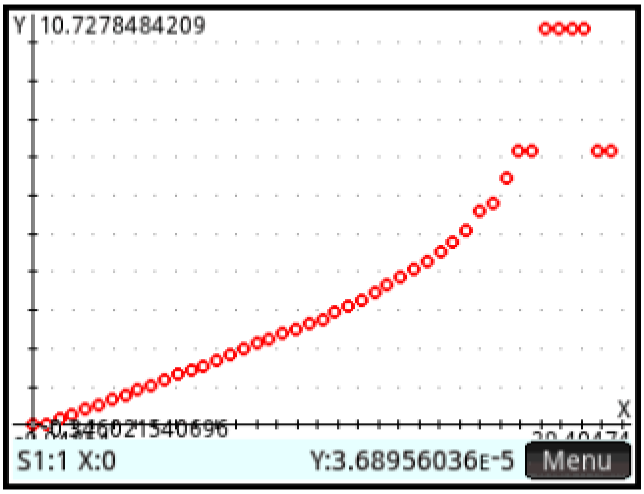


For warming ($\boldsymbol{k > 0}$), the next formula applies for temperature $T(t)$ at time *t*:

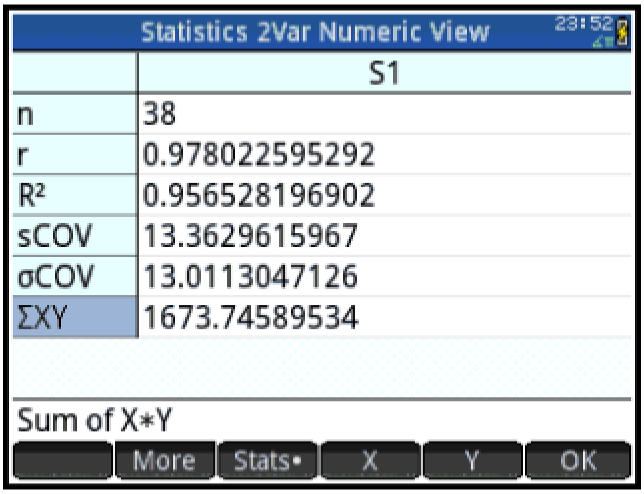
$$\boldsymbol{T(t) = T_{\text{hot}} + (T_{\text{start}} - T_{\text{hot}}) \cdot e^{-k \cdot t}} \leftrightarrow$$

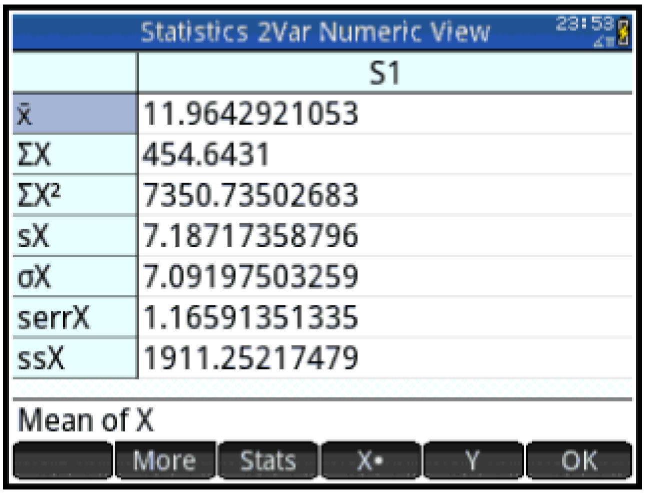
$$\boldsymbol{-\ln\left[\frac{T(t) - T_{\text{hot}}}{T_{\text{start}} - T_{\text{hot}}}\right] = k \cdot t}$$

T_{hot} is the temperature of the hot environment, which measured 84.19 °C in our experiment. In this case, we explore the relationship between $y \equiv -\ln\left[\frac{T(t) - T_{\text{hot}}}{T_{\text{start}} - T_{\text{hot}}}\right]$ versus *t* (*x* axis), which is a line through the origin with a positive slope *k*:



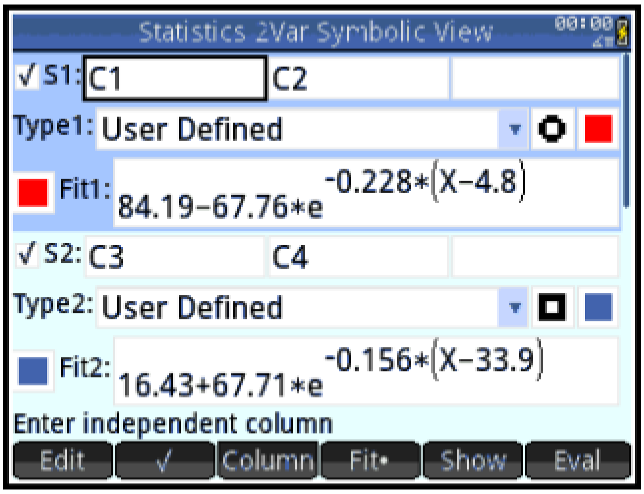
Ignoring the last seven points, we calculate the slope $k = \frac{\sum x \cdot y}{\sum x^2}$:



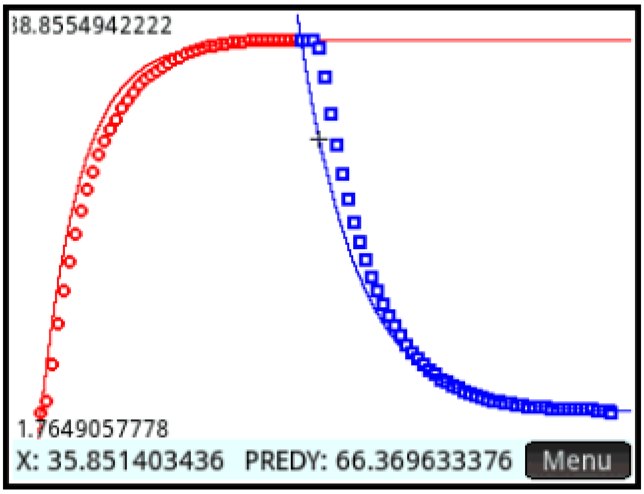


Now, we have also modelled the warming process: $\boldsymbol{T(t) = 84.19 - 67.76 \cdot e^{-0.228\cdot(t-4.80)}}$.

Finally, we compare the experimental data with the numerical models by entering them as user-defined fits in the Statistics 2Var app, using the Symb[oblic View] key, and plot them:



The largest difference between the empirical data and our numerical models appears at point no. 49, where the measured value amounts to 83 °C but the predicted one is 66 °C in the graph. Yet, the graph shows that the experimental data approximately follow Newton's law of warming and cooling.



Appendix: Experiment data

No.	Warming	
	t (in s)	*T* (in °C)
1	4.80	16.43
2	5.44	18.51
3	6.09	25.38
4	6.74	32.50
5	7.38	38.55
6	8.03	43.98
7	8.68	48.90
8	9.33	53.32
9	9.97	57.10
10	10.62	60.23
11	11.27	63.32
12	11.91	65.88
13	12.56	68.05
14	13.21	69.89
15	13.85	71.82
16	14.50	73.43
17	15.15	74.84
18	15.79	76.05
19	16.44	77.06
20	17.09	77.95
21	17.73	78.72
22	18.38	79.40
23	19.03	79.97
24	19.67	80.61
25	20.32	81.12
26	20.97	81.63
27	21.61	82.04
28	22.26	82.45
29	22.91	82.76
30	23.55	83.06
31	24.20	83.27
32	24.85	83.47
33	25.49	83.63
34	26.14	83.78
35	26.79	83.93
36	27.43	83.98
37	28.08	84.09
38	28.73	84.14
39	29.37	84.14
40	30.02	84.19
41	30.67	84.19
42	31.31	84.19
43	31.96	84.19

No.	Cooling	
	t (in s)	*T* (in °C)
46	33.90	84.14
47	34.55	84.09
48	35.19	84.09
49	35.84	83.01
50	36.49	77.59
51	37.13	70.89
52	37.78	65.09
53	38.43	59.72
54	39.07	55.32
55	39.72	51.14
56	40.37	47.53
57	41.01	44.23
58	41.66	41.22
59	42.31	38.52
60	42.96	36.17
61	43.60	34.03
62	44.25	32.11
63	44.90	30.41
64	45.54	28.80
65	46.19	27.40
66	46.84	26.16
67	47.48	25.07
68	48.13	24.11
69	48.78	23.27
70	49.42	22.54
71	50.07	21.88
72	50.72	21.29
73	51.36	20.81
74	52.01	20.36
75	52.66	19.93
76	53.30	19.55
77	53.95	19.19
78	54.60	18.89
79	55.24	18.63
80	55.89	18.39
81	56.54	18.18
82	57.18	18.00
83	57.83	17.86
84	58.48	17.71
85	59.12	17.59
86	59.77	17.47
87	60.42	17.38
88	61.06	17.29

44	32.61	84.14
45	33.25	84.14

89	61.71	17.20
90	62.36	17.17
91	63.00	17.11
92	63.65	17.06
93	64.30	17.00
94	64.94	16.97
95	65.59	16.94
96	66.24	16.91
97	66.88	16.85
98	67.53	16.85
99	68.18	16.82
100	68.50	16.43