EXERCISES
HP Prime
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HP Prime Calculator

- **Switch on calculator**: Press \( \text{On} \).
- **Switch off calculator**: Press \( \text{Shift} \) and then \( \text{On} \).
- **To select the „degree“ mode**:
  - Open the configuration window by pressing \( \text{Shift} \) \( \text{Settings} \).
  - Select Degrees or Radians using F2 (CHOICE).
- **To select the complex number regime**:
  - Use the drop down menu and select enter in algebraic form \( a+ib \) or injure using two real numbers \((a,b)\).
- **To access the calculator controls**:
  - All calculator controls are grouped in the list accessible by pressing \( \text{D} \).
- **For access to special symbols**:
  - The calculator offers a truly large number of symbols accessible by pressing \( \text{Shift} \) \( \text{Vars} \).
Optimization: Area of a Triangle

**HP Prime**

**Level:** First year of French Lyceum (the 10th year of obligatory schooling in France)

**Objective:** An introduction to functions, their graphs and written form
The maximum Stating assumptions using dynamic geometry.

**Keywords:** functions, tables, values, showing graphs, maximum.

**Problem:** Let A be a point located at the vertex opposite the base of an isosceles triangle. Point C lies on a circle centered at A whose radius is \([AB]\). Find the location of C that will maximize the area of the triangle ABC.

**Step-by-step solution:**
The HP Prime Calculator is used to graph geometry problems and make use of the dynamic possibilities of the „Geometry“ application by pressing I.

For access to sketches, press Plot. The individual menus of the „Geometry“ application allow the construction of triangles and circles. The point C will be placed as an active point.
The placement of individual geometric objects on the display may be confirmed by pressing Enter.

Access to individual geometric elements which have been drawn and their titles may be had by pressing Y.

The area of the triangle and the length of its base may be calculated by pressing M.

We make use of the command buttons labelled area.

We will shift the position of the point C and for each location of C, record the resulting area.
In this way, we may obtain a number of value pairs (base; area), which may be stored in a table. Select the application „Statistics 2Var“ by pressing $x^2$.

Each pair of values (base; area) is entered into the table (by pressing $M$).

By pressing $P$, we obtain the corresponding point graph which shows that the points describe a curve with an extremum, here a maximum.

The graph reveals that the area should be at a maximum when the length of the base is equal to 10.8.

An analytical solution may also be chosen to discover the algebraic form of the function which expresses the area of the triangle in dependence upon the length of the base $|BC| = x$. The height $AH$ must be expressed as a function of $x$. 

![Diagram of a triangle with labeled points A, B, C, and H, and a value 8 associated with the height.](image)
The Pythagorean theorem for the right triangle AHC is:

\[ |AH| = \sqrt{8^2 - \frac{x^2}{4}}. \]

The area of the triangle ABC, then, is given by the formula

\[ \frac{x \cdot \sqrt{64 - \frac{x^2}{4}}}{2}. \]

We enter this expression in the HP Prime Application: „Function“ and press \( Y \).

By pressing \( P \), we obtain the graph of the expression.

Using the \( \text{Fct} \rightarrow \text{Extremum} \) you get to the curve's maximum point.
The „Grazing Goat“ Problem

HP Prime

Level: First year of French Lyceum (the 10th year of obligatory schooling in France)

Exercise: A shepherd has a square-shaped pasture with a 10 m circumference. He ties the goat to a line anchored to a post located at the midpoint of one side of the square. He wishes to have the goat graze an area equal to one half of the area of the pasture. How long must the line be to which the goat is tied?

Step-by-step solution:
The HP Prime Calculator is equipped with a “Geometry” application which enables the situation to be illustrated graphically.

Press \( \boxed{\text{Y=}} \) and select the „Geometry“ icon.

Construct the square using the Polygon > Special > Square menu.

Screenshots:
Place the centre of the circle at the midpoint of the upper boundary of the square to delineate the area grazed by the goat.

Use the „Midpoint“ tool in the Point menu. Then select „Circle“ in the „Curve“ menu and draw the requisite circle.

Then position an active point on the inner semi-circle of the square and designate the radius starting from that point, which symbolizes the rope to which the goat is tied.

Subsequently, you can either increase or decrease the circle radius (and thereby the length of the line).
If the length of the line is shorter than the side of the square pasture, the surface the goat can graze equals a semi-circle whose radius is given by the length of the rope.

If the length of the line is longer than the side of the square, the area consists of a rectangle and a circular segment.

To determine the width of the rectangle, use the algebraic form of the Pythagorean theorem for the right triangle in the opposite screenshot: \( x^2 = 5^2 + \text{width}^2 \)

Width of the rectangle = \( \sqrt{x^2 - 25} \)

To calculate the area under the arc, we deduct the area of the red triangle from the area of the sector:

\[
\frac{\alpha}{360} \pi x^2 - \frac{10 \sqrt{25 - x^2}}{2}
\]

is the angle of the centre, which is calculated using the goniometric function \( 2 \arcsin(\frac{5}{x}) \).

Subsequently, we can write a program to calculate the area of the pasture the goat grazes as a function of line length:

```
EXPORT KOZA()
BEGIN
LOCAL L;
// We require the length of the line
INPUT(L);
// we process both cases of the surface area
IF L<=5 THEN
PRINT(\pi*L*L/2);
ELSE
PRINT(\sqrt{(L*L-25)*10+2*\arcsin(5/L)/360*\pi*L*L-5*\sqrt{(L*L-25)}});
END;
END;
```

Make sure you set the unit of angular measure to degrees.

Button: ```
After entering the data into the program, the result shows that a 50 m² area = 100 m² ÷ 2 would have a line length of approximately 5.8 m.
Metal Rods and Springs

HP Prime

Problem:
Rigid metal rods AC of 4 cm, BD of 7 cm and CD of 18 cm are placed so that the CD rod is horizontal and the AC and BD rods are perpendicular to it.
An active point M is located on the rod CD. The point M is connected to point A using a spring and to point B by another spring.
Determine the position of point M that minimizes the sum of the spring lengths.

Step-by-step solution:
The HP Prime Calculator is equipped with a „Geometry“ application which enables the situation to be illustrated.
Press \( \text{Home} \) and select the „Geometry“ icon.

The configuration indicated above may be illustrated by selecting „Segment“, enabling the point M to move along the horizontal section. The line sections designate both springs.

Screenshots:
If we move the point M, the spring will dynamically follow.

By pressing $Y$, we obtain access to all geometric objects.

The $M$ button initiates calculations for the various objects. This may be used to calculate the lengths via the distance button. In our case, the distance (GH,GJ) is calculated as the distance between the points GH and GJ, i.e., the distance corresponding to the length of the first spring. The second distance calculation corresponds to the length of the second spring.
Move point M from the graphic window (button P) and return to the number menu (button M) to determine the change in the lengths.

Let there be a rod [CD] of constant length of 18 cm, vertical rod [AC] of 4 cm and a vertical rod [BD] of constant length of 7 cm and let the length CM be the x variable. Using the algebraic expression of the Pythagorean theorem you get:

\[ AM = \sqrt{16 + x^2} \quad \text{and} \quad BM = \sqrt{(18 - x)^2 + 49} \]

The sum of both spring lengths can be entered like this in the „Function“ application in the HP Prime calculator (by pressing then „Function“, then Y).

By pressing , you get the graph and minimum value for the length of both springs for \( x = 6.5 \). Then you get the position of point M to achieve the minimum total spring length: M must be \( 6.5 \text{ cm} \) from point C.
Varignon Parallelogram

HP Prime

1/ Make a hypothesis about the type of quadrilateral with vertices at the midpoints of the four sides of any quadrilateral.
2/ Prove the hypothesis.
3/ Designate the type of quadrilateral if the external quadrilateral is a rectangle.

Step-by-step solution:
1/ The dynamic geometry of the HP Prime calculator is accessible using the \( \boxed{\text{I}} \) button.

Draw any quadrilateral using the menu \( \boxed{\text{Polygon}} \) > „Quadrilateral“.

Screenshots:
Position the first vertex of the quadrilateral by touching any point on the display and confirm it by pressing Enter. Repeat this operation for the other three vertices.

Now position the midpoints using the Point menu >"Midpoint" by pressing both edge points on each side of the quadrilateral. After each selection of edge point, press Enter.

Using the function Polygon > Quadrilateral draw an inscribed quadrilateral following the procedure described in the previous case.

**Useful trick:** The inscribed quadrilateral may be filled with colour by pressing Z and selecting „Fill with Color“ and selecting the quadrilateral you have just constructed.

It seems that the inscribed quadrilateral is a parallelogram.
The HP Prime calculator can verify this. To have it do so, first select the name of the parallelogram by pressing \( \mathbf{Y} \). In this case, the parallelogram is named GQ (name for a geometric object).

Then press \( \text{Prefs} \) and select \( \text{is\_parallelogram} \) in the menu. Enter the name of the quadrilateral in parentheses:

\[
\text{is\_parallelogram(GQ)}
\]

and press OK.

The HP Prime displays the result:
0 if it is not a parallelogram
1 if it is a parallelogram
2 if it is a rhombus
3 if it is a rectangle
4 if it is a square

In this case, HP Prime displays 1: the inscribed quadrilateral is a parallelogram.

\[
\text{is\_parallelogram(GQ)}: 1
\]

2/ This is easy to prove using the theorem on centres applied to both triangles of the external quadrilateral which are separated by a diagonal.

\[
MN = \frac{1}{2}AC \quad \text{et} \quad PO = \frac{1}{2}AC \quad \text{donc} \quad MN = PO
\]

This means the quadrilateral MNOP is a parallelogram.
3/ Let us require the external quadrilateral to be a rectangle. To do so, enter the coordinates of all four starting points using the menu:

- **GA**: point(-4,2)
- **GB**: point(4,2)
- **GC**: point(4,2)
- **GD**: point(4,-3)
- **GE**: quadrilateral(GA, GB, GC, GD)
- **GG**: segment(GA, GB)
- **GH**: segment(GB, GC)
- **GI**: segment(GC, GD)
- **GJ**: segment(GD, GA)
- **GK**: midpoint(GA, GB)

The inscribed quadrilateral is thus a rhombus.
Maximum amount of chocolate

HP Prime

A supermarket purchases boxes of chocolates for a unit price of € 5 from a chocolate factory for Christmas. The supermarket sells one box for €13.6. Last year, 3000 boxes were sold during the same period of time. Market research shows that each 10 euro-cent reduction in price results in increased sales of 100 boxes of chocolates per week. Help the supermarket to determine the price per box to attain a maximum profit.

You can hand out work sheets indicated on page 24 to your students.

Step-by-step solution:
1/ Access to the „Spreadsheet“ of the HP Prime calculator is via I.

Create a table of values with automated formulas using a €0.00 discount of the sales price followed by stepwise €0.10 discounts. First fill in the individual column headers by entering the following names in the cells: DISCOUNT, PRICE, BOXES, SALES a PROFIT. To do so, go to A in the first column and enter DISCOUNT using the following alphabetical characters:

AqAcAdAtAF

and press Name in the menu.

Carry out this operation for all columns.
To enter the discount values, go to DISCOUNT and enter the following formula:

\[ S.N. \times S.A.Q \]

The entire column will be filled in with an arithmetic sequence with a constant difference of 0.1 between its members.

Now enter the price formula by going to PRICE and entering:

\[ S.x.z.v.w \]

To enter the boxes, go to BOXES and enter the formula indicated in the image on the right.

For boxes go to SALES and enter the formula indicated in the image on the right.

For boxes go to PROFIT and enter the formula indicated in the image on the right.

Thus you obtain referential links to the names of columns in formulas.

All resulting calculations will now be automatically displayed.

In the table, we will work top-down in order to observe the evolution of profits.

We discover that the maximum profit is obtained when we sell one box for €10.80.
Useful trick: If you wish to colour certain cells, place the cursor on them, press Format > Color and select the colour from the menu.

Now you can test using a function. If x stands for the sales price per single box, profit is expressed as follows:

\[(x - 5) \cdot (3000 + (13.6 - x) \cdot 1000)\]

Enter the expression in the „Function“ application (I) in the symbolic depiction window (Y):

Use `IFTE` to get the graph of the function. Its extremum (here a maximum) may be obtained by pressing Fcn > Extremum. It will be confirmed, once again, that the maximum profit of €33,640.00 can be obtained if the boxes with chocolates are to be sold at a unit price of €10.80.

Useful trick: you can press `IFTE` to apply the SI (IF) condition located in the table processor. This command button is used as follows: `IFTE` (if, then, otherwise), see the problem indicated on the right.
Chocolates: Student Worksheet

HP Prime

Calculate the supermarket’s purchase price and weekly profit for 3000 boxes of chocolates sold at a unit price of €13.60:

Calculate the profit made if the price per box of chocolates is reduced by €0.10.

Fill in the table:

<table>
<thead>
<tr>
<th>Number of discounts</th>
<th>Price (€)</th>
<th>Boxes (€)</th>
<th>Sales (€)</th>
<th>Profit (€)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>13.60</td>
<td>3000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>13.50</td>
<td>3100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Create a table using the table processor, activate it and fill it in to determine the maximum attainable profit.
Creating an **HP Prime** Program

To integrate and process algorithms using the HP Prime calculator use a program editor.

**Step-by-step solution:**

Access to the program editor of the HP Prime calculator is via Shift 1. A list of programs saved in the calculator will be displayed. To create a new program press New.

Enter the name of the program.

![New Program](image)

Enter new name

The program should be written between the BEGIN and END commands.

```plaintext
BEGIN
ALGO();
END;
```
Creating an Notation/Notebook

**HP Prime**

Entering text is not a program and therefore cannot be run. Only text may be formatted and saved in the HP Prime calculator's memory.

**Step-by-step solution:**
You can access the HP Prime text (note) editor by pressing \( \text{Shift} \) \( \text{Num Lock} \). Press \( \text{New} \) to create a new program.

The text may be formatted using the Style and Format tabs. The text may be formatted with bold, italics, underlined, crossed, colour (foreground colour) and highlighted (background colour). To do so, select the colour of your choice from the corresponding palette.

To browse the list you can use indents.

**Screenshots:**

**Equation of tangent**

Equation of a tangent to a curve representing a function \( f \):

- \( y = f'(a)(x-a) + f(a) \)

To browse the list you can use indents.
Basic Algorithms and Loops on the

HP Prime

Level: First year of French Lyceum (the 10th year of obligatory schooling in France)

Objectives: Algorithms have been included in the mathematics syllabus for secondary schools. Algorithms start to be taught in the first to second year of secondary schools which corresponds to the first year of the French secondary school system or the 10th year of the French obligatory schooling system. Here is a selection of algorithms taught at French secondary schools:

Step-by-step solution:
Problem 1: the first/basic algorithm
Write an algorithm requiring you to enter a number $x$ and displaying a transcription of function $f(x) = x^2 + 6x - 4$.

Algorithm

Enter
Ask the user to transcribe function

Processing
Save the function transcription $x^2 + 6x - 4$ in the $y$ variable

Output
Display $y$

Screenshots:
Make a note in HP Prime:

```
EXPORT ALGO1()
BEGIN
INPUT(X);
X^2+6*X-4→Y;
PRINT(Y);
END;
```
Second problem: the „For“ loop:
Write an algorithm requiring you to enter the initial values of $n$ which calculates the factorial of this number.

Algorithm
Enter
Request the user to provide the initial number
Initialization
Enter 1 in the $P$ variable
Processing
For $i$ in the interval 1 to $n$
Save
$P^i$ in $P$
End of for $i$ loop
Output
Display $p$

Third problem: „Until“ loop:
Find the largest integer $p$ such that the sum of integers 1 to $p$ is lower than the given integer $n$.

Use formula (1 ES / S):
\[
\sum_{k=1}^{p} k = \frac{p(p + 1)}{2}
\]

Algorithm
Enter
Request the user to provide a number $n$
Initialization
Enter 1 in the $P$ variable
Processing
Until $P^*(P+1)/2$ is lower than $n$
Save $P+1$ in $P$
End of Until loop
Output
Display

Write in HP Prime

```
EXPORT ALGO2()
BEGIN
1>P;
FOR I FROM 1 TO N DO
P*I>P;
END;
PRINT(P);
END;
```

Write in HP Prime

```
EXPORT ALGO3()
BEGIN
INPUT(N);
1>P;
WHILE P*(P+1)/2<=N DO
P+1>P;
END;
PRINT(P);
END;
```
Algorithm: Heron’s Formula

HP Prime

Heron's formula allows to calculate the area of the triangle:

\[ A = \sqrt{p(p-a)(p-b)(p-c)} \]

where \( p \) is half of the triangle's circumference.

Program an algorithm to calculate the area of a triangle using Heron’s formula.

Step-by-step solution:

Using the editor (press \[ \text{Shift} \left[ \times \right] \)), we will create the HERON program and write the following algorithm:

```plaintext
EXPORT HERON()
BEGIN
LOCAL A,B,C,P;
//The user is requested to enter the side lengths for all sides of the triangle
INPUT(A);
INPUT(B);
INPUT(C);
// Calculate half the perimeter of the triangle(A+B+C)/2 ➔ P;
//Calculate the area of the triangle using Heron's formula
PRINT(√(P*(P-A)*(P-B)*(P-C)));
END;
```

For the values \( a = 2 \), \( b = 7 \) and \( c = 8 \), the program displays:

Screenshots:

![Screenshot of HP Prime calculator displaying the HERON program and its output](image-url)
Algorithm: BMI Calculator

HP Prime

BMI (Body Mass Index) is an indicator used to assess the health or lack of health of one's weight (obesity). BMI primarily enables the assessment of overweight or obesity. Calculation of the BMI provides only basic information, because the calculation does not take into account bone weight or muscle weight.

BMI is calculated using the formula:

$$\frac{P}{T^2}$$

Let P be the body weight in kilograms and T the height in meters.

The World Health Organization (WHO) has developed the following category system:

<table>
<thead>
<tr>
<th>WHO Classification</th>
<th>BMI value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Underweight</td>
<td>&lt; 18.5</td>
</tr>
<tr>
<td>Normal weight</td>
<td>18.5 – 24.9</td>
</tr>
<tr>
<td>Overweight</td>
<td>25 – 29.9</td>
</tr>
<tr>
<td>Moderate obesity (Class I)</td>
<td>30 – 34.9</td>
</tr>
<tr>
<td>Grave obesity (Class II)</td>
<td>35 – 39.9</td>
</tr>
<tr>
<td>Morbid obesity (Class III)</td>
<td>≥ 40</td>
</tr>
</tbody>
</table>

Create an algorithm to calculate the BMI and classify the result using the WHO classification.
Step-by-step solution:
Using the editor, we will create the BMI program and write the following algorithm:

```
EXPORT BMI()
BEGIN
  LOCAL P,T,I;
  //The user is requested to enter his weight and height INPUT(P,"Your weight in kg:");
  INPUT(T,"Your height in m:");
  // Calculate BMI
  P/T^2►I;
  //Calculate classification
  IF I<18.5 THEN PRINT("BMI="+I+" underweight"); END;
  IF I≥18.5 AND 24.9≥I THEN PRINT("BMI="+I+" normal weight"); END;
  IF I≥25 AND 29.9≥I THEN PRINT("BMI="+I+" overweight"); END;
  IF I≥30 AND 34.9≥I THEN PRINT("BMI="+I+" moderate obesity (Class I)"); END;
  IF I≥35 AND 39.9≥I THEN PRINT("BMI="+I+" serious obesity (Class II)"); END;
  IF I≥40 THEN PRINT("BMI="+I+" morbid obesity (Class III)"); END;
END;
```

Screenshots:

```
EXPORT BMI()
BEGIN
  LOCAL P,T,I;
  //The user is asked to enter his body weight
  INPUT(P,"Your weight in kg:");
  INPUT(T,"Your height in m:");
  //We will calculate BMI
  P/T^2►I;
  //Perform classification
  IF I<18.5 THEN PRINT("BMI="+I+" underweight"); END;
  IF I≥18.5 AND 24.9≥I THEN PRINT("BMI="+I+" normal weight"); END;
  IF I≥25 AND 29.9≥I THEN PRINT("BMI="+I+" overweight"); END;
  IF I≥30 AND 34.9≥I THEN PRINT("BMI="+I+" moderate obesity (Class I)"); END;
  IF I≥35 AND 39.9≥I THEN PRINT("BMI="+I+" serious obesity (Class II)"); END;
  IF I≥40 THEN PRINT("BMI="+I+" morbid obesity (Class III)"); END;
END;
```

BMI=25.390625 overweight
Algorithm: „Secret Number“ Game

HP Prime

Program an algorithm which requires the user to find a random whole number in the interval between 1 and 100, with each test specifying whether the number entered is higher or lower than the secret number.

**Step-by-step solution:**
Using the editor, we will create the MYSTERE (SECRET) program and write the following algorithm:

```plaintext
EXPORT MYSTERE()
BEGIN
LOCAL M,N;
//Choose a random whole number between 1 and 100
1+FLOOR(100*RANDOM) ► N;
//The user is requested to enter a number
INPUT(M);
//The user is continually requested to enter a new number until the number corresponds to the secret number, with information provided as to whether the number entered is higher or lower than the secret number
WHILE M<>N DO
    IF M>N THEN
        MSGBOX("Lower");
    ELSE
        MSGBOX("Higher");
    END;
    INPUT(M);
END;
MSGBOX("Secret number found!");
END;

The MSGBOX button is similar to the PRINT but except that it shows the text in a dialog window rather than an output window.

**Screenshots:**

<table>
<thead>
<tr>
<th>Program Catalog</th>
<th>131.59</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALGO</td>
<td>0KB</td>
</tr>
<tr>
<td>ALGO1</td>
<td>&lt;1KB</td>
</tr>
<tr>
<td>ALGO2</td>
<td>&lt;1KB</td>
</tr>
<tr>
<td>ALGO3</td>
<td>&lt;1KB</td>
</tr>
<tr>
<td>BMI</td>
<td>1KB</td>
</tr>
<tr>
<td>Function (App)</td>
<td>0KB</td>
</tr>
<tr>
<td>HERON</td>
<td>1KB</td>
</tr>
<tr>
<td>MYSTERE</td>
<td>1KB</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Program Catalog</th>
<th>131.61</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALGO</td>
<td>0KB</td>
</tr>
<tr>
<td>ALGO1</td>
<td>&lt;1KB</td>
</tr>
<tr>
<td>ALGO2</td>
<td>&lt;1KB</td>
</tr>
<tr>
<td>ALGO3</td>
<td>&lt;1KB</td>
</tr>
<tr>
<td>BMI</td>
<td>1KB</td>
</tr>
<tr>
<td>Function (App)</td>
<td>0KB</td>
</tr>
<tr>
<td>HERON</td>
<td>1KB</td>
</tr>
<tr>
<td>MYSTERE</td>
<td>1KB</td>
</tr>
</tbody>
</table>
```
Algorithm: Calculate the Greatest Common Divisor (GCD) by Subtraction

HP Prime

Program an algorithm which will show the individual steps in calculating the greatest common divisor (GCD) using the subtraction method.

Step-by-step solution:
Using the editor, we will create the SOUST (SUBTRACTION) program and write the following algorithm:

```
EXPORT SOUST()
BEGIN
LOCAL A,B,C;
//The user is requested to enter two positive integers from which to calculate the GCD
INPUT(A);
INPUT(B);
PRINT(A+" ; "+B);
//Take the smaller of the selected numbers and the difference between the larger and smaller number
MIN(A,B) ► C;
MAX(A,B)−MIN(A,B) ► B;
C ► A;
PRINT(A+" ; "+B);
//Take the smaller number once again and the difference between the numbers until you get an equal value
WHILE A<>B DO
MIN(A,B) ► C;
MAX(A,B)−MIN(A,B)−MIN(A,B) ► B;
C ► A;
PRINT(A+" ; "+B);
END;
//Display the GCD value
PRINT (C);
END;
```

Screenshots:
Algorithm: Calculation of the Greatest Common Divisor (GCD) – Euclid’s Algorithm

HP Prime

Program an algorithm which will show the individual steps in calculating the greatest common divisor (GCD) using Euclid’s algorithm.

Step-by-step solution:

Use the editor (press Shift \(\times\)) to create the program EUC and enter the following algorithm:

```
EXPORT EUC()
BEGIN
LOCAL A,B,C;
// The user is requested to enter two positive integers for which he
wishes to calculate the greatest common divisor (GCD)
INPUT(A);
INPUT(B);
PRINT(A+" ; "+B);
// Take the smaller number of the two numbers entered and the
remainder from dividing the greater number by the smaller
MIN(A,B) ► C;
irem(MAX(A,B),MIN(A,B)) ► B;
C ► A;
PRINT(A+" ; "+B);
// Take the smaller number and the remainder once again until it is
not equal to zero
WHILE B<>0 DO
  MIN(A,B) ► C;
  irem(MAX(A,B),MIN(A,B)) ► B;
  C ► A;
  PRINT(A+" ; "+B);
END;
// Display the GCD value
PRINT(C);
END;
```

Screenshots:
Algorithm: Magic Trick

HP Prime

Program an algorithm which requires the user to find a random whole number in the interval between 1 and 100, with each test. The magician asks a spectator:

- Think of a number.
- Multiply it by two.
- Subtract 3.
- Multiply it by 6.
- Tell me the result.

Create a SPECT (SPECTATOR) program which will display the number the spectator told the magician and a MAGIE (MAGIC) program which will, based upon the result announced, find the number the spectator thought of, specifying whether the number entered is higher or lower than the secret number.

Step-by-step solution:
Using the editor, we will create the EUC program and write the following algorithm:

```plaintext
EXPORT SPECT()
BEGIN
LOCAL N;
//Ask a spectator to enter a number he is thinking of
INPUT(N);
//Carry out the calculations requested by the magician and display
PRINT((2*N-3)*6);
END;

EXPORT MAGIE()
BEGIN
LOCAL N;
//Enter the number announced by the spectator
INPUT(N);
//Run the calculation program and move stepwise in reverse Display the result which is the number the spectator was thinking of
PRINT(((N/6+3)/2));
END
```

For instance, say the spectator was thinking of 18. After carrying out the requested operations, the spectator announces 198. MAGIE will look up 18 for the input number of 198.
Algorithm: Leap Year

HP Prime

Leap years are those years which are:
• either divisible by 4 but not divisible by 100,
• or divisible by 400.

Write a program which will determine whether a particular year is a leap year.

Step-by-step solution:
Using the editor (press \textit{Shift} \textit{X}), we will create the EUC program (press \textit{Shift} \textit{Y}), and write the following algorithm:

\begin{verbatim}
EXPORT BISS()
BEGIN
LOCAL N;
//The user is requested to enter a year
INPUT(N);
//Check the conditions for leap years
IF (irem(N,4)==0 AND irem(N,100)<>0) OR irem(N,400)==0
THEN
PRINT(N+" this is a leap year");
ELSE
PRINT(N+" this is not a leap year");
END;
END
\end{verbatim}

To be able to use the „Which day of the year were you born on?“ algorithm, enter the input directly in the name of the program and the output will be replaced by 1 for leap years or 0 for non-leap years:

\begin{verbatim}
EXPORT BISS(N)
BEGIN
IF (irem(N,4)==0 AND irem(N,100)<>100) OR irem(N,400)==0
THEN
RETURN(1);
ELSE
RETURN(0);
END
\end{verbatim}

\begin{verbatim}
1900 is not a leap year.
2016 is a leap year.
\end{verbatim}

\begin{verbatim}
BISS(1984) 1
BISS(2007) 0
\end{verbatim}
Algorithm: Which Day of the Year Were You Born on?

HP Prime

This method allows you to determine the day of the week for a particular date in the interval between 1900 and 2099:

- A number code 033 614 625 035 (January = 0, February = 3, etc.) will be assigned to each month of the year.
- Add: the number created by the two last numbers of the year in which the person was born, a quarter of this number (rounded down if it is not an integer), date of the day of birth (i.e., an integer between 1 and 31) and the month code.
- If the date of birth occurred after 2000 subtract 1 from the result.
- If it is a leap year and the date of birth is before March 1, subtract 1 from the result.
- Divide by 7 and the quotient determines the day of the week (0 = Sunday, 1 Monday, etc.).

Write a program whose result will be the day of the week on which you were born.

Step-by-step solution:
Using the editor, we will create the JOUR (DAY) program and write the following algorithm:

```plaintext
EXPORT JOUR()
BEGIN
LOCAL A,M,J,N,P,L1,L2;
// The user is request to enter his date of birth
// The user is requested to enter the year
INPUT(A,“Year?”);
// The user is requested to enter the month
INPUT(M,“Month (from 1 to 12)?”);
// The user is requested to enter the day
INPUT(J,“Day (from 1 to 31) ”);
// create a list containing the codes of months in the year
{0,3,3,6,1,4,6,2,5,0,3,5}►L1;
// If the year of birth is after 2000 subtract 1
0►P;
IF A>2000 THEN P-1►P; END;
// If it is a leap year and a month before March, subtract 1
IF BISS(A)==1 AND M<3 THEN P-1►P; END;
// Remove last two digits in the year
irem(A,100)►A;
// Carry out the calculation described in the assignment information
A+FLOOR(A/4)+J+L1(M)+P►N;
// To determine the day of the week, divide by 7
{“Sunday”,“Monday”,“Tuesday”,“Wednesday”,“Thursday”,“Friday”,
“Saturday”}►L2;
irem(N,7)►N;
PRINT(“You were born on”L2(N+1)); END;
```

Screenshots:

You were born on Sunday
Contour Line Method

HP Prime

Level: First year of French Lyceum (the 10th year of obligatory schooling in France)

Exercise: In the Cartesian coordinate system, locate all points for whose coordinates \((x, y)\) the following is true
\[x \cdot (6 - x) < y \cdot (8 + y)\].

Step-by-step solution:
The HP Prime programs include the „Advanced Graphing” application which is so powerful that no programming is necessary for this exercise.

Press \(\text{Menu}\) and go to „Advanced Graphing”.

Next to \(V1\) enter the inequality for the exercise:
\[x \cdot (6 - x) < y \cdot (8 + y)\]
By pressing \( \text{Shift} \) \( \text{Plot} \) you set the axis scale: set the X coordinates between -10 and 10 and Y coordinates between -10 and 10.

Press \( \text{Plot} \) to display the graph.

The HP Prime displays a graph with the corresponding points.
**Friday the 13th**

**HP Prime**

**Level:** First year of French Lyceum (the 10th year of obligatory schooling in France)

**Exercise:** Demonstrate that there is at least one Friday the 13th every year.

**Programming themes:** loops, conditions, use of lists.

---

**Step-by-step solution:**

Create three lists: one list for all the days in the week (Monday, Tuesday, etc.), one for all the months in the year and one list for the number of days in the month.

Then let's take January 13 as the basis. Depending upon whether the date falls on Monday, Tuesday, Wednesday, Thursday, Friday, Saturday or Sunday, look whether there is a Friday the 13th. You do so by gradually checking all the subsequent months.

To display the result, add the number of days in a month to the input day and calculate the remainder of this sum using Euclid's algorithm (Euclidean division by 7). If the remainder after the division is 5, it means Friday (Friday is the 5th day in the week).
Subsequently, create the following program in HP Prime:

```hp
EXPORT V13()
BEGIN
  LOCAL L1,L2,L3,I,J,M;
  PRINT;
  L1:={"Monday","Tuesday","Wednesday","Thursday","Friday","Saturday","Sunday"};
  L2:={"January","February","March","April","May","June","July","August","September","October","November","December"};
  FOR I FROM 1 TO 7 DO
    PRINT("If it is 13 January "+L1(I)+":");
    I►M;
    I►J;

    WHILE irem(J,7)≠5 AND M<12 DO
      J+L3(M)►J;
      M+1►M;
      END;
    IF irem(J,7)==5 THEN
      PRINT("13. "+L2(M)+" it is Friday 13th");
    ELSE
      PRINT("does not include Friday 13th");
    END;
  END;
END;
```

The result of this program shows that regardless of what day in the week the 13th January falls upon, a Friday the 13th will always follow.
Kaprekar’s Constant

HP Prime

Kaprekar’s Constant is a number whose square root may be divided into a left and right side (with a value not equal to 0) whose sum equals the initial number.

Example: $4879^2 = 23804641$ a $238 + 04641 = 4879$.

Create an algorithm which determines whether a particular number qualifies as Kaprekar’s Constant.

Programming themes: loops, conditions, use of lists.

Step-by-step solution:

First extract the root of each digit of the square root of the number selected. Save each number in a list. To extract the root of individual numbers use Euclid’s algorithm and carry out gradual Euclidian division by 10, taking each remainder from the division. The REVERSE command button (allowing you to reverse the list to display the numbers as they are written from left to right in the final square root of the selected number).

To create the list, all combinations of left and right sides must be tested. To obtain all the combinations, both For loops will overlap. Note the numbers obtained for each side by using multiplication by 10. After both sides are created, carry out an equality test. If Kaprekar’s equality has been proven (the sum of both sides equals the initial number), the display will show the number is an example of Kaprekar’s Constant (potentially providing a detailed decomposition). If the equality is not proven, no information is displayed.
We enter the following in the HP Prime:

```plaintext
EXPORT KAPREKAR()
BEGIN
INPUT(N);
N→Z;
L1:={};
N*N→N;
WHILE N≠0 DO
  CONCAT(L1,{irem(N,10)}) → L1;
  iquo(N,10) → N;
END;
REVERSE(L1) → L1;
FOR I FROM 1 TO SIZE(L1)−1 DO
  0 → G;
  0 → D;
  FOR J FROM 1 TO I DO
    G*10+L1(J) → G;
  END;
  FOR J FROM I+1 TO SIZE(L1) DO
    D*10+L1(J) → D;
  END;
  IF G+D==Z THEN
    PRINT(Z+" est un nombre est de Kaprekar.");
    PRINT(Z+"²=Z*Z" et "Z=+G+++D);
  END;
END;
END;
```

The program may be tested, e.g., using 703, which meets the requirements for Kaprekar’s Constant.

703 is a number of Kaprekar.  
703²=494209 and 703=494+209
Algorithm: Birth Limitation

HP Prime

Level: The first year of French Lyceum (the 10th year of obligatory schooling in France)

Objectives: Verifying the hypothesis, writing and the use of an algorithm.

Keywords: Probability, algorithm, iteration, while loop.

Task: A certain country restricts the number of births of girls so that:
• Each family can have a maximum of 4 children.
• After the birth of a boy the family must not have more children.

What is the impact of this policy on fertility of the local population?

Step-by-step solution:
We perform a simulation using the following algorithm, which indicates the frequency of the occurrence of a boy:

Variables:
N: Number of families
G: Total number of boys
F: Number of girls in the same family
E: Number of children in the same family
T: Total number of births

Processing:
Insert N
Initialise G at 0
Initialise T at 0
For I, which varies from 1 to N
  Initialise E at 0
  Initialise F at 0
  While E<4 do
    Select a random integer S between 1 and 2
    E will have a value E+1
    T will have a value T+1
    If S=1
      Then G will have a value G+1
    In the opposite case F will have a value F+1
  End if
  End While
End For

Output:
Print G/T
End

Screenshots:
If we run the algorithm for a large number of families, the frequency of boys is very close to 0.5. This implies that this birth rate policy has probably no effect on the number of boys.

It can be demonstrated that the population policy will not change anything, if we create branching probabilities and calculate the probabilities:

The results can be summarised in the following table:

<table>
<thead>
<tr>
<th>Number of children N</th>
<th>Number of boys G</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0</td>
<td>1/16</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1/16</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1/8</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1/4</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1/2</td>
</tr>
</tbody>
</table>
These probabilities can be derived:
\[ E(N) = 4 \times \frac{1}{16} + 4 \times \frac{1}{16} + 3 \times \frac{1}{8} + 2 \times \frac{1}{4} + 1 \times \frac{1}{2} = \frac{15}{8} \]
\[ E(G) = 1 \times \frac{1}{16} + 1 \times \frac{1}{8} + 1 \times \frac{1}{4} + 1 \times \frac{1}{2} = \frac{15}{16} \]
and \[ \frac{E(-G)}{E(N)} = \frac{1}{2}. \]

By pressing the \( W \) button we get the exact value in the form of a fractional notation.
The Caesar cipher principle is that each letter is replaced with the letter that is three places further down the alphabet (A is replaced with D, B is replaced with E, C is replaced with F, etc.). The word SECRET is encrypted as VHFUHW.

1/ Create an algorithm to encode a specific word using the Caesar cipher.

2/ Create an algorithm to decode a specific word that is encrypted using the Caesar cipher.

---

**Step-by-step solution:**

The HP Prime Calculator has very useful command buttons that enable the processing and taking out of characters from a character string:

- The **LEFT** or the **RIGHT** command button selects groups of characters at the beginning or end of a character string.
- The **MID** command button allows you to take out any character from a character string.
- By using the **SIZE** command button, it is possible to calculate the number of characters in the character string. Character strings are placed in quotation marks.

The **ASC** command button changes the ASCII code of the character string. This button can be used in order to obtain the position of a certain letter in the alphabet.

**CHAR** is the opposite command button. It performs direct return of the letter from its ASCII code. These two command buttons are in this case very practical because there is no need to use a list consisting of all the letters of the alphabet in the algorithm.

---

**Screenshots:**

LEFT("Bonjour",1)  "B"
RIGHT("Bonjour",2)  "ur"
MID("Bonjour",3,1)  "n"
SIZE("Bonjour")  7

ASC("A")  [65]
ASC("B")  [66]
ASC("A")-64  [1]
ASC("T")-64  [20]
CHAR(65)  "B"
CHAR(20+64)  "T"
1/ With these useful command buttons the following algorithm can be performed on the HP Prime calculator:

```
EXPORT CESAR()
BEGIN
  //We locally enter the lower case variable n
  local n;
  LOCAL S,M,K;
  ""M;
  //The user is asked to enter the word that is to be encrypted
  INPUT(n,"Insert in quotation marks","The word that is to be en-
  crypted");
  SIZE(n)S;
  FOR K FROM 1 TO S DO
    //Each letter is shifted by three positions and the encrypted word
    is generated
    M+CHAR(ASC(MID(n,K,1))+3)M;
  END;
  PRINT(M);
END;
```

2/ Now we will decrypt the encrypted word. We proceed in the reverse direction:

```
EXPORT CESAR()
BEGIN
  local n;
  LOCAL S,M,K;
  ""M;
  //The user is asked to enter the encrypted word
  INPUT(n,"Insert in quotation marks","Encrypted word");
  SIZE(n)S;
  FOR K FROM 1 TO S DO
    //This time we shift by 3 letters backwards
    M+CHAR(ASC(MID(n,K,1))-3)M;
  END;
  PRINT(M);
END;
```
Sicherman Dice

Sicherman Dice are a pair of 6-sided dice: sides of the first dice are numbered 1, 2, 2, 3, 3 and 4; sides of the other dice are numbered 1, 3, 4, 5, 6 and 8. If we roll these 2 dice and add up the results of the sides, not only will we have the same options as in the case of the classic dice (2 to 12), but also the same frequency of occurrence!

Create a programme that rolls Sicherman dice as well as classic dice five hundred times each and compare the frequency of resulting totals using a chart.

**Step-by-step solution:**

We store the totals of both sides obtained for both types of dice and 500 rolls each (For loop from 1 to 500) into two lists - L3 and L4.

RANDINT(1,6) gives a random integer between 1 and 6.

EXPORT SICHERMAN()
BEGIN
LOCAL L1,L2,L3,L4;
L1:={1,2,2,3,3,4};
L2:={1,3,4,5,6,8};
L3:={};
L4:={};
FOR I FROM 1 TO 500 DO
CONCAT(L3,{RANDINT(1,6)+RANDINT(1,6)})L3;
CONCAT(L4,{L1(RANDINT(1,6))+L2(RANDINT(1,6))})L4;
END;
END;

**Screenshots:**

[Chart showing frequency of resulting totals for Sicherman and classic dice rolls.]

EXPORT SICHERMAN()
BEGIN
LOCAL L1,L2,L3,L4;
L1:={1,2,2,3,3,4};
L2:={1,3,4,5,6,8};
L3:={};
L4:={};
FOR I FROM 1 TO 500 DO
CONCAT(L3,{RANDINT(1,6)+RANDINT(1,6)})L3;
CONCAT(L4,{L1(RANDINT(1,6))+L2(RANDINT(1,6))})L4;
END;
END;
If you want to statistically use both lists created by the programme, it is necessary to save them into variables D1 and D2.

Then we start the “Statistics 1Var” application, that can be accessed by pressing the I button.

The two lists created using the programme will be displayed in the first 2 columns.

We press the Y button to select the chart we want to display. We select the graphic representation in the form of a histogram and insert D2 into the H2 field. First, we select D1, which displays the histogram obtained using normal dice.
We display the histogram by pressing the \textcircled{P} button.

Now select H2 and press the \textcircled{Y} button again.

By pressing the \textcircled{P} button we obtain the histogram for rolls with Sicherman dice. The histogram has the same shape as the histogram of the normal dice.

The higher the number of rolls, the more the Sicherman dice histogram will be close to the histogram obtained by normal dice rolls.
Lottery Draw

HP Prime

Create a programme simulating a lottery draw (5 numbers from 1 to 49, and 1 lucky number from 1 to 10).

**Step-by-step solution:**

In this case, the problem lies in the fact that we cannot draw a ball which has already been drawn. Therefore, it is necessary to create a list containing all 49 drawn balls. After each draw we will remove the drawn ball from the list using the `remove` command button.

The use of the HP Prime calculator is very easy, compared with the programming of a lottery draw without restoring it to its original state using a complex spreadsheet processor or compared with what some other brand calculators offer.

The `MAKELIST` command button makes it easy to create a list of 49 integers from 1 to 49.

Into the HP Prime calculator you just need to write:

```plaintext
EXPORT LOTTERY()
BEGIN
    MAKELIST(N,N,1,49,1)L1;
    49N;
    FOR I FROM 1 TO 5 DO
        L1(RANDINT(1,N))B;
        PRINT(B);
        remove(B,L1)L1;
        N−1N;
    END;
    PRINT("Lucky number: "+RANDINT(1,10));
END;
```

**Screenshots:**

![Lottery Draw Screenshot](image)

49
28
20
6
16
N° chance : 6
**Plotting of a Spiral**

**HP Prime**

**Level:** The first year of French Lyceum (the 10th year of obligatory schooling in France)

**Task:** Plot a spiral generated by plotting half-circles centred alternately at point O and point A.

![Image of a spiral plot]

**Step-by-step solution:**

We build 20 half-circles starting with a semicircle of radius 5.

HP Prime will draw circular arcs using the command button `ARC_P(x,y,R,a1,a2,C)`, where (x, y) are the coordinates of the centre, R is the radius, a1 and a2 specify the angle defined by the arc and C its colour. If we want to successively change centres of the half-circles from point O to point A, we add to the original x coordinate the remainder of the remaining radii after the Euclidean division by twice the radius. So we will successively add 0 or the radius. Half-circles will display successively with differences of angles between 0 and $\pi$ and then between $\pi$ and $2\pi$. Then it can be used in the loop incremented in I values $(I-1)\pi$ and $I\pi$.

`RECT_P();` allows you to view a clear window before displaying. `FREEZE;` stops the screen on the drawing.

```
EXPORT SPIRAL()
BEGIN
    RECT_P();
    FOR I FROM 1 TO 20 DO
        ARC_P(150+irem(5*I,10),120,5*I,p*(I-1),p*I,RGB(255,0,0));
    END;
    FREEZE;
END;
```

**Screenshots:**

![Screenshot of the spiral plot]
Random Walk

HP Prime

The flea, that we initially place on an axis with a scale, will carry out 1000 consecutive jumps. Each time it jumps, it will randomly shift forward or backward by a certain unit, without a preferred direction of movement forward or backward. Plot the route the flea will travel.

Step-by-step solution:

We will draw a random number 0 or 1, in order to know whether the flea jumps forward or backward. In a loop, we perform 1000 jumps and after each stage we display, using the pixel coordinates, whole consecutive numbers from 1, and using the ordinal position of the flea after the jump.

On the same diagram, it is possible to simulate several random walks so that we introduce a loop 1 to 5 (to display 5 curves) and we colour differentiate individual curves using an RGB code that we make dependent on the variable incrementing of the loop.

We write into HP Prime:

```plaintext
EXPORT FLEA()
BEGIN
LOCAL I,J,P,X,Y;
RECT_P;
FOR I FROM 1 TO 5 DO
P:=0;
FOR I FROM 1 TO 1000 DO
IF RANDINT(0,1)==0 THEN
P:=P+1;
ELSE
P:=P-1;
END;
I:=X;
P:=Y;
PIXON_P(X,100+Y,RGB(255-40*J,40+50*J,215));
END;
END;
FREEZE;
END;
```

Screenshots:
Combination of Cards in Poker

**HP Prime**

**Task:** In a poker game, we get combination of 5 cards by random selection from a pack of 32 cards. Create a programme showing a combination of cards in a poker game.

**Step-by-step solution:**

We create one list with card values and a second list with card suits (clubs, spades, hearts, diamonds). For this list we can use special characters of the HP Prime calculator. The calculator offers four card suits (buttons Shift and ).

The HP Prime calculator is equipped with a large number of graphic command buttons. These buttons can be easily used to draw cards (rectangles) and display card values and suits in the two corners as with actual playing cards.

We can write the following programme:

```plaintext
EXPORT POKER()
BEGIN
LOCAL I,L1,L2,M,N,H;
RECT_P();
L1:={"1","R","D","V","9","8","7"};
L2:={"♥","♦","♠","♣"};
FOR I FROM 1 TO 5 DO
RECT_P(15+60*(I−1),50,15+60*(I−1)+50,130,RGB(255,235,200));
RANDINT(1,4) H;
IF H<3 THEN
255 N;
ELSE
0 → N;
END;
RANDINT(1,7) M;
TEXTOUT_P(L1(M),18+60*(I−1),51,RGB(N,0,0));
TEXTOUT_P(L2(H),15+60*(I−1),64,RGB(N,0,0));
END;
FREEZE;END;
```

**Screenshots:**

![Screenhots of HP Prime calculator showing a poker game program](image)
Simulation Programmes

HP Prime

This programme is very useful for teaching probability. It allows the simulation of random experiments below:

• Tossing a coin (heads or tails)
• Rolling a 6-sided die
• Wheel of fortune
• Drawing balls from a lottery drum
• Drawing cards
• Random numbers

The Student Worksheet can be used in teaching.

Step-by-step solution:
The below stated programme simulates the above mentioned experiments using diagrams. The HP Prime calculator is equipped with a wide graphical menu of command buttons for easy performance of these simulations.

Copy the following programme to the programme editor (buttons Shift Vars A x Vars A Vars A):

EXPORT ProbaSim()
BEGIN
    //Press the ESC button to quit the current simulation
    //First open the Statistics 1Var application and only then start the ProbaSim programme
    LOCAL C,R,I1,I2;
    D1:={};
    D2:={};
    I1:=0;
    I2:=0;
    L1:={"HEADS","TAILS"};
    L2:={195,195,115,115};
    L3:={70,150,150,70};
    L4:={#00C617h,#FFD800h,#0094FFh,#FF0000h,#CE0059h};
    //Select the required simulation type
    CHOOSE(C, "Select simulation", "Coins (Heads or Tails) ", "6-sided die", "Wheel", "Lottery drum", "Cards", "Random numbers");

Screenshots:
/Each time you press the ENTER button, the programme will turn the coin and display “HEADS” (PILE) or “TAILS” (FACE).
The HP Prime calculator is very quick, if you hold the ENTER button for 10 seconds, it will carry out 150 tosses
IF C==1 THEN
WHILE ISKEYDOWN(4)<1 DO
RECT;
ARC_P(155,110,80,0,360,RGB(124,78,41));
IF ISKEYDOWN(30)==1 THEN
R:=1+FLOOR(RANDOM(2));
TEXTOUT_P(L1(R),140,105,3);
I1:=I1+1;
END;
WAIT;
END;
ELSE

//Rolling of a 6-sided die is simulated by displaying a square on which a whole, randomly drawn number (between 1 and 6) is written.
IF C==3 THEN
WHILE ISKEYDOWN(4)<1 DO
RECT;
ARC_P(155,110,80,0,360,RGB(124,78,41));
LINE_P(155,30,155,190);
LINE_P(75,110,235,110);
TEXTOUT_P("1",192,55);
TEXTOUT_P("2",195,152);
TEXTOUT_P("3",113,155);
TEXTOUT_P("4",110,55);
IF ISKEYDOWN(30)==1 THEN
R:=1+FLOOR(RANDOM(4));
LINE_P(155,110,L2(R),L3(R),RGB(255,0,0));
I1:=I1+1;
D1:=CONCAT(D1,{I1});
TEXTOUT_P("Draw No."
+I1,130,200,1);
D2:=CONCAT(D2,{R});
END;
WAIT;
END;
ELSE
//The wheels of fortune simulation displays a hand that randomly falls into one of the four quarters of the circle.

IF C==2 THEN
WHILE ISKEYDOWN(4)<>1 DO
RECT;
RECT_P(115,70,195,150,2,RGB(255,194,124));
IF ISKEYDOWN(30)==1 THEN
R:=1+FLOOR(RANDOM(6));
TEXTOUT_P(R,153,102,3,RGB(210,0,0));
I1:=I1+1;
D1:=CONCAT(D1,(I1));
TEXTOUT_P("Draw No."+I1,130,200,1);
D2:=CONCAT(D2,{R});
END;
END;
ELSE
//For the lottery drum, we draw a lottery drum and a coloured tablet (a random draw - a selection from 5 colours)
IF C==4 THEN
WHILE ISKEYDOWN(4)<>1 DO
RECT;
ARC_P(155,110,50,135,405,RGB(0,135,234));
LINE_P(190,75,215,40,RGB(0,135,234));
LINE_P(120,75,95,40,RGB(0,135,234));
IF ISKEYDOWN(30)==1 THEN
R:=1+FLOOR(RANDOM(5));
RECT_P(135,40,170,60,3,L4(R));
I1:=I1+1;
D1:=CONCAT(D1,(I1));
TEXTOUT_P("Draw No."+I1,130,200,1);
D2:=CONCAT(D2,{R});
END;
WAIT;
END;
ELSE

The last two simulations “Cards” and “Random numbers” are described in the separate tutorials “Combination of Cards in Poker” and “Lottery Draw”

```
IF C==5 THEN
  WHILE ISKEYDOWN(4)<1 DO
    IF ISKEYDOWN(30)==1 THEN
      RECT_P();
      L1:={'1','R','D','V','9','8','7'};
      L2:={'♥','♦','♠','♣'};
      FOR I FROM 1 TO 5 DO
        RECT_P(15+60*(I−1),50,15+60*(I−1)+50,130,RGB(255,235,200));
        H:=RANDINT(1,4);
        IF H<3 THEN
          N:=255:
        ELSE
          N:=0:
        END;
        M:=RANDINT(1,7);
        FOR I FROM 1 TO 5 DO
          TEXTOUT_P(L1(M),18+60*(I−1),51,3,RGB(N,0,0));
          TEXTOUT_P(L1(M),55+60*(I−1),115,3,RGB(N,0,0));
          TEXTOUT_P(L2(H),15+60*(I−1),64,3,RGB(N,0,0));
          TEXTOUT_P(L2(H),52+60*(I−1),100,3,RGB(N,0,0));
        END;
      END;
    END;
    WAIT;
  END;
ELSE
  IF C==6 THEN
    WHILE ISKEYDOWN(4)<1 DO
      PRINT;
      IF ISKEYDOWN(30)==1 THEN
        L1:=MAKELIST(N,N,1,49,1);
        N:=49:
        FOR I FROM 1 TO 5 DO
          B:=L1(RANDINT(1,N));
          PRINT(B);
          remove(B,L1);
          N:=N-1:
        END;
        PRINT("Lucky number: "+RANDINT(1,10));
      END;
      WAIT;
    END;
  END;
ELSE
  END;
END;
```

```
N° chance : 5
```

On the HP Prime calculator, perform 100 simulations for each experiment and complete the following table:

<table>
<thead>
<tr>
<th>Experiment:</th>
<th>Frequency of occurrence of coin sides</th>
<th>Decimal value of the frequency of occurrence of coin sides</th>
<th>Probability of tossing tails</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coins (Heads or Tails)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experiment:</td>
<td>The frequency of occurrence of 6</td>
<td>Decimal value of the frequency of occurrence of 6</td>
<td>Probability of rolling 6</td>
</tr>
<tr>
<td>Die</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experiment:</td>
<td>The frequency of occurrence of 3</td>
<td>Decimal value of the frequency of occurrence of 3</td>
<td>Probability of spinning 3</td>
</tr>
<tr>
<td>Wheel of fortune</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experiment:</td>
<td>The frequency of occurrence of yellow</td>
<td>Decimal value of the frequency of occurrence of yellow</td>
<td>Probability of drawing yellow</td>
</tr>
<tr>
<td>Lottery drum</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experiment:</td>
<td>The frequency of occurrence of</td>
<td>Decimal value of the frequency of occurrence</td>
<td>Probability</td>
</tr>
<tr>
<td>Cards</td>
<td>Heart:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cards</td>
<td>Ace:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cards</td>
<td>Ace of Hearts:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experiment:</td>
<td>The frequency of occurrence of</td>
<td>Decimal value of the frequency of occurrence</td>
<td>Probability</td>
</tr>
<tr>
<td>Random numbers</td>
<td>7:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Random numbers</td>
<td>1 as a lucky number:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Random numbers</td>
<td>Two consecutive numbers:</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Each company in France has a unique identification number SIRET (Système d'Identification du Répertoire des Etablissements/Register of companies).

The SIRET code consists of 14 digits, the last digit is a check digit. SIRET is formed in the following way:

- Each code digit is placed from position 14 to position 1.
- The digits in the odd positions are multiplied by 1 and the digits in the even positions by 2.
- We add up the digits of each multiplication result.
- We add up the results of individual positions.

If the result is a multiple of 10, the SIRET code is valid.

Example: The Ministry of Education SIRET: 11004301500012

| 14 | 1 | 3 | 12 | 1 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
|----|---|---|----|---|----|---|---|---|---|---|---|---|---|---|---|
| N= | N= | M= | M= | Q= | P= | M= | N= | R= | M= | M= | M= | N= | O= |
| 1x2 | 1x1 | 0x2 | 0x1 | 4x2 | 3x1 | 0x2 | 1x1 | 5x2 | 0x1 | 0x2 | 0x1 | 1x2 | 2x1 |

2+1+0+0+8+3+0+1+1+0+0+0+2+2 = 20, this is a multiple of 10.

Create a control algorithm for the SIRET code.

**Step-by-step solution:**

The user is asked to enter the SIRET code.

HP Prime is able to process 12-digit numbers at the maximum. Therefore, the request made by the user must be divided into two: the first 12 digits and then the last two.

The programme with explanatory notes:

```plaintext
EXPORT SIRET()
BEGIN
  INPUT(M,"The first 12 digits of the SIRET code");
  INPUT(N,"The last two digits of the SIRET code");
  L1:={};
  //We store the first 12 digits to a list
  FOR I FROM 1 TO 12 DO
    irem(M,10) R;
    iquo(M,10) M;
    CONCAT(L1,{R}) L1;
  END;
  //We add to them the last two entered digits
  CONCAT(L1,{irem(N,10),iquo(N,10)}) L1;
END;
```

**Screenshots:**

```plaintext
EXPORT SIRET()
BEGIN
  INPUT(M,"The first 12 digits of code SIR");
  INPUT(N,"The last two digits of code SIR");
  L1:={};
  //Save the first 12 digits to list
  FOR I FROM 1 TO 12 DO
    irem(M,10) R;
    iquo(M,10) M;
    CONCAT(L1,{R}) L1;
  END;
  //We add the last two entered digits
  CONCAT(L1,{irem(N,10),iquo(N,10)}) L1;
END;
```
We multiply all the digits in the even positions by 2
FOR I FROM 1 TO 7 DO
L1(2*I)*2 P;

// If the result contains more than one digit, each digit is added
DIM(STRING(P)) L;
IF L>1 THEN
FOR J FROM 1 TO L DO
D+irem(P,10) D;
iquo(P,10) P;
END;
ELSE
E+P E;
END;
END;
0 S;
// We make a sum of digits in the odd positions
FOR I FROM 0 TO 6 DO
S+L1(2*I+1) S;
END;
// We check if the resulting sum is a multiple of 10
IF irem(D+E+S,10)==0 THEN
PRINT("The SIRET code is valid");
ELSE
PRINT("The SIRET code is invalid");
END;
END;

We insert the SIRET code (twice: the first 12 digits and then the last 2) and the programme will show whether the SIRET code is valid or invalid.
**ISBN Code**

**HP Prime**

Each issued book is identified by a unique ISBN code (International Standard Book Number). The ISBN code consists of 10 digits; the last digit is a check digit.

The code can be verified as follows: We add the first nine digits after multiplying each digit by its position number. The remainder of the weighted sum of all digits divided by 11 must be the check digit (the last digit).

Note: If the check digit is 10, it’s written using an X.


<table>
<thead>
<tr>
<th>ISBN</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Digit</td>
<td>2x1</td>
<td>5x2</td>
<td>0x3</td>
<td>1x4</td>
<td>0x5</td>
<td>8x6</td>
<td>6x7</td>
<td>9x8</td>
<td>0x9</td>
<td></td>
</tr>
</tbody>
</table>

2 + 10x0 + 4x0 + 48 + 42 + 72 + 0 = 178 = 11x16 + 2 and 2 is therefore the last digit.

**Step-by-step solution:**

The user is asked to enter the ISBN code (10 digit code). The programme with explanatory notes:

```
EXPORT ISBN()
BEGIN
LOCAL I,R,S;
INPUT(N);
L1:={}; //We save each ISBN digit into a list
FOR I FROM 1 TO 10 DO
irem(N,10) R;
iquo(N,10) N;
CONCAT(L1,{R}) L1
END; //We change the order of digits in the list so they have the same order as in ISBN
REVERSE(L1) L1;
//We add up multiples of the first 9 digits, which we got by multiplying each ISBN digit by their positions in the code
0 S;
FOR I FROM 1 TO 9 DO
S+L1(I)*I S;
END; //We check whether the remainder of the weighted sum of all digits divided by 11 is the last digit
IF irem(S,11)==L1(10) THEN
PRINT("The ISBN code is valid");
ELSE
PRINT("The ISBN code is invalid");
END;
END;
```

**Screenshots:**

The ISBN code is valid
**Algorithm: Matchsticks Game**

**HP Prime**

This game is a game for two players. We start the game with 10 matches. Players may alternately remove 1 to 3 matches. The player who removes the last match is the loser. Create a programme that allows you to play this game.

**Step-by-step solution:**
The programme with explanatory notes:

```
EXPORT MATCH()
BEGIN
LOCAL N,J,M,X,Y,I;
//We first determine the number of matches at 10 and the first player is set to 1
10 N;
1 J;
//The players take turns, until there is only one match left
WHILE N>1 DO
    INPUT(M,"Player"+J,"How many matches do you want to remove?");
    IF M>3 THEN
        MSGBOX("Maximum of 3 matches!");
    ELSE
        IF J==1 THEN 2 J; ELSE 1 J;
        END;
        N-M N;
        END;
    MSGBOX("There remains"+N+" matches");
    END;
//A notification, which player lost
MSGBOX("Player"+J+" lost!");
END;
```

**Screenshots:**

[Image of the HP Prime calculator interface with the programme running]
**Bonus:** The programme can be improved by creating a graphical interface:

EXPORT MATCH()
BEGIN
LOCAL N,J,M,I;
10 N 1 J;
//We draw 10 rectangles that represent the matches
RECT_P;
TEXTOUT_P("Player"+J,10,10,1,1);
FOR I FROM 1 TO 10 DO
RECT_P(10+20*I,30,25+20*I,50,3,RGB(186,0,0));
RECT_P(10+20*I,50,25+20*I,122,3,RGB(181,135,83));
END;
//We display matches for 5 seconds
WAIT(5);
//The players take turns, until there is only one match left
WHILE N>1 DO
INPUT(M,"Player"+J,"How many matches do you want to remove?");
IF M>3 THEN
MSGBOX("Maximum of 3 matches!");
ELSE
IF J==1 THEN 2 J; ELSE 1 J;
END;
N-M N;
END;
//We display the remaining matches
RECT_P;
TEXTOUT_P("Player"+J,10,10,1,1);
FOR I FROM 1 TO N DO
RECT_P(10+20*I,30,25+20*I,50,3,RGB(186,0,0));
RECT_P(10+20*I,50,25+20*I,122,3,RGB(181,135,83));
END;
WAIT(5);
END;
//A notification, which player lost
MSGBOX("Player"+J+" lost!");
END;
Algorithm: Spaghetti Exercise

HP Prime

I have one spaghetti noodle. What is the probability that when the spaghetti is cut into three pieces, I can construct a triangle with these 3 pieces?

Step-by-step solution:
In this case, we verify the triangle inequality using three randomly obtained lengths of spaghetti.
We write the total length of the spaghetti noodle.
We can create the following algorithm:

Algorithm
Input
We enter the number of trials N
We enter the spaghetti length L

Initialisation
Initialisation of the variable R (number of successful solutions)

Processing:
For I = 1 to N
Cut the first piece of length X
(X = random number of type 0 < X < L)
Cut the second piece of length Y
(Y = random number of type 0 < Y < L–X)
Calculate the length of the third piece Z (Z = L–X–Y)
If the maximum of these three lengths is less than or equal to the sum of the two remaining lengths
Thus increase R by 1
End „If“
End„For“

Output
Print R/N

The algorithm specifies the frequency of occurrence of triplets using verification of the triangle inequality.
The higher the number of trials, the more the frequency approaches sought probability.
Algorithm: Bouncing Ball
HP Prime

We bounce a ball from the initial height of 300 cm. We assume that with each bounce from the ground the ball loses 10% of its height (with each bounce the height is multiplied by 0.9). Find out how many bounces from the ground are necessary for the height of the ball to be less than or equal to 10 cm. Write an algorithm to solve this task.

Step-by-step solution:
We will gradually reduce the previous height by 10% of the original height until we reach the height of 10 cm. In the algorithm, we use the loop „While“:

Algorithm

Initialisation
Number h initialised at the value of 300
Number n initialised at the value of 0

Processing:
While h > 10
Save h*0.9 in h
Save n+1 in n
End of the While loop

Output
Print n
Weight: Gravitational Force

HP Prime

Duration: 1 hour

Objective: Reaction of a weight to gravitational force, an introduction to gravitational acceleration and familiarisation with the formula $F = mg$

Equipment: HP Prime, StreamSmart, dynamometer, scales

Task: Measuring weights of different objects of different masses using a force sensor (dynamometer).

Step-by-step solution:
First we set the force sensor to +10N. We weigh the object first and then we hang it on the hook of the sensor. We start obtaining data in the DataStreamer application to measure the force in Newtons (N).

If we hang, for example, an HP Prime calculator (which weighs 224 g = 0.224 kg), the sensor displays value -2.60 N.

We weigh other objects (such as another three new generation HP calculators) to get the following table:

<table>
<thead>
<tr>
<th>Object</th>
<th>Weight (kg)</th>
<th>Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HP Prime</td>
<td>0.224</td>
<td>2.60</td>
</tr>
<tr>
<td>HP 39gII</td>
<td>0.249</td>
<td>2.61</td>
</tr>
<tr>
<td>HP 300S+</td>
<td>0.146</td>
<td>1.89</td>
</tr>
<tr>
<td>HP 10S+</td>
<td>0.122</td>
<td>1.61</td>
</tr>
</tbody>
</table>

Screenshots:

Canal 1 Force

Duration: 1 hour

Objective: Reaction of a weight to gravitational force, an introduction to gravitational acceleration and familiarisation with the formula $F = mg$

Equipment: HP Prime, StreamSmart, dynamometer, scales

Task: Measuring weights of different objects of different masses using a force sensor (dynamometer).
We insert this table to the „Statistics 2Var“ application (the \( \text{Menu} \) button).
We add row 0 N for 0 kg.

We set the regression to a linear type (the \( \text{Y} \) button).

Points are more or less aligned (the \( \text{Plot} \) button).

By pressing the \( \text{Y} \) button we obtain characteristics of the straight line.
The passing of the straight line through the beginning can be written using the equation \( y = 10x \). This means that \( F = m \cdot g \), where \( F \) is the weight expressed in N, in relation to \( m \) expressing the weight in kg.
\( g \) is the slope of the straight line (approx. 10). This is the so called gravitational acceleration (which in fact has a value of about 9.81 N/kg).
Sound Waves

HP Prime

**Duration:** 1 hour

**Objective:** Characterise the type of a sinusoidal sound wave based on music played on a piano keyboard.

**Equipment:** HP Prime, StreamSmart, microphone, keyboard, loudspeaker

**Task:** Measure the period and calculate the frequency of the first seven notes played on a piano keyboard. Determine the type of the bass tone sound wave. You can use the Student Worksheet.

**Step-by-step solution:**
On the piano keyboard (if you have a computer with speakers you can use a virtual keyboard, which can be downloaded from the Web: [http://www.bgfl.org/custom/resources_ftp/client_ftp/ks2/music/piano/](http://www.bgfl.org/custom/resources_ftp/client_ftp/ks2/music/piano/)) we will play the first 7 tones, and we record each tone using a microphone that is connected to the StreamSmart application.

When we press the DataStreamer application will display a real-time audio recording done by a microphone.
The picture opposite shows the first part of the recording of the first keyboard note. The curve resembles a sinusoid.

The sinusoidal curve is more apparent after export and zoom. The sound wave from the piano spreads through the air between the speaker and the microphone. The wave is mechanical, gradual and periodic because the curve represents periodic function of time: The undulation repeats itself in equal intervals of time.

The frequency and period are related and this relationship can be expressed by the equation: \( f = \frac{1}{T} \).

We measure the period (a time interval between two peaks of the sinusoid): 0.015 s, representing a frequency of about 67 Hz. It means „C“ of the first octave.

In the following tone we observe a shorter period (sine curve segments are shorter): 0.0135 s, i.e., frequency of 74 Hz, which corresponds to the „D“ note of the first octave.

Low tones have a low frequency, while the high notes have a high frequency.
The tone frequency in the next octave is twice as big (e.g. „C“ of the second octave has frequency 2x67 = 134 Hz).
The opposite picture shows a curve, which we obtained by pressing the last keyboard key. The period is very short (very short sections of the sinusoid). The tone is very high.

When we look at the bass tone (we select DOUBLE BASS on the virtual keyboard), we get a curve which doesn’t have a sinusoidal shape. It contains several overtones.
Sound waves: Student Worksheet

HP Prime

Fill in the lines and the table below:
The shape of curves observed in the StreamSmart application:

Definition of periodic gradual mechanical waves:

<table>
<thead>
<tr>
<th>Keyboard key</th>
<th>Period (s)</th>
<th>Frequency (Hz)</th>
<th>Musical tone</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Establish a link between the frequency and tone height (high or low):

Compare the frequency of the same note in the same octave, and the frequency of the same note one octave higher:
Objective: Perform a test measurement and familiarise yourself with the concepts of relative humidity and atmospheric pressure.

Equipment: HP Prime, StreamSmart, thermometer, hygrometer, barometer

Task:
1/ Perform simultaneous measurements of the air pressure, air temperature and ambient humidity.
2/ Interpret the air pressure based on the current weather.
3/ Analyse the table below showing humidex (heat index) values and give each colour an explanatory legend
Step-by-step solution:

1/ Use the three sensors (thermometer, barometer and hygrometer) which you connect to the StreamSmart application at the same time, the DataStreamer application will display results of all three measurements in real time.

2/ In our example, the measured values will be constant. Therefore it is not necessary to display curves. Using the button, we will display only values measured by each sensor. Channel 3 shows that the ambient pressure is 101.61 kPa. This means that although it doesn’t rain, the weather could be bad! Even at high atmospheric pressure it may be cloudy. Lower pressure encourages the rising of the air containing water droplets (ambient humidity is 68.32%, indicating the presence of water in the air) which gather and then fall as precipitation. Humidity is 68.32 %. Atmospheric humidity is expressed as a percentage and represents the ratio between the amount of water in the air and the maximum amount of water that the air can contain. If we measure relative humidity of 50%, it means that the air contains half the amount of the maximum amount of water vapour that it can contain.

We measured ambient temperature at 21.42 °C.

3/ For relative humidity of 70% and a temperature of 21 °C, the field of the heat index table is blue and displays 25. The value of 25 corresponds to the felt temperature (in °C). Blue fields indicate an acceptable felt temperature. Green fields indicate some discomfort. Yellow fields indicate great discomfort when it is necessary to restrict strenuous physical activity. Orange fields indicate danger. Red fields indicate high risk (heatstroke) with a possible risk to life. The heat index can be interpreted as a measure of comfort.
Blood Spots

HP Prime

When teaching practical and scientific methods, it is possible, in particular, to use examples of forensic criminology. In this experiment we will analyse blood spots that were found at a crime scene, and the analysis will establish the link between the diameter of the spots and the height from which they fell.

**Equipment:** HP Prime, ink, blank sheets of paper, meter

**Experiment:**
1/ Let drops of ink fall from different heights onto large sheets of blank paper.

2/ For each height, calculate the mean droplet diameter after impact.

3/ Enter data into the HP Prime calculator and perform regression to establish the link between the height and the diameter of drops of blood.

4/ We found drops of blood with the mean value diameter of 19 mm, left by a killer who is bleeding from his head. How high is the killer?

**Step-by-step solution:**

Sample results of the experiment:

<table>
<thead>
<tr>
<th>Height (cm)</th>
<th>The mean value of the diameter (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>6.8</td>
</tr>
<tr>
<td>50</td>
<td>13.4</td>
</tr>
<tr>
<td>100</td>
<td>17</td>
</tr>
<tr>
<td>150</td>
<td>17.9</td>
</tr>
<tr>
<td>200</td>
<td>20</td>
</tr>
</tbody>
</table>

We enter the data into the calculator using the „Statistics 2Var“ application.
We can have a look at the graphical representation by pressing the \(\text{Plot}\) button and then the \(\text{View}\) button for the automatic scale selection.

The HP Prime calculator will directly perform regression (the picture opposite shows a quadratic regression).

By pressing the \(Y\) button we set the type. We test each type of regression to find the most accurate one (the curve which passes closest to all points).

The logarithmic regression is the most suitable.
By pressing the \( Y \) button again, we get the values of regression coefficients:

Equation: \( f(x) = 4.32 \ln(x) - 3.25 \)

Now we can enter this expression to the „Function“ application and display the value corresponding to 19 mm to find the perpetrator’s height in cm.

The killer is approx 1.73 m.
Traces of Blood: Student Worksheet

HP Prime

Explain how the diameter of the drops of blood is changing depending on the height from which they fall:

<table>
<thead>
<tr>
<th>Height of fall (cm)</th>
<th>Drop diameter 1 (mm)</th>
<th>Drop diameter 2 (mm)</th>
<th>Drop diameter 3 (mm)</th>
<th>Mean drop diameter (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Specify the type of regression, which allows obtaining a representative curve of the mean diameter of drops of blood, depending on the height from which the drops fall:

Determine the height of the killer:
Box Plot

HP Prime

Box plot is a graphical representation which consists of:
- The „box“ part of the diagram whose upper and lower ends indicate the first and third quartiles,
- Two horizontal lines (whiskers) outside the box connecting the minimum value and the first quartile on one side, and the third quartile and maximum value on the other side,
- The vertical band inside the box is the median.

Create a box plot for the following statistical series:
78; 79; 77; 59; 57; 65; 65; 68; 67; 59; 54; 64; 68; 72; 74; 72; 72; 76; 77; 76; 74; 77; 76

**Step-by-step solution:**

We start the „Statistics 1Var“ application by pressing the `I` button.

We insert the series values in the first column of the table which we can access by pressing the `M` button.

**Screenshots:**

Application Library

- Function
- Advanced Graphing
- Geometry
- Spreadsheet
- Statistics 1Var
- Statistics 2Var
- Inference
- DataStreamer

Save Reset Sort Send Start

Statistics 1Var Numeric View

<table>
<thead>
<tr>
<th></th>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>D4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>78</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>79</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>3</td>
<td>77</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>4</td>
<td>59</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>5</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
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<td></td>
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<tr>
<td>7</td>
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<td>8</td>
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<tr>
<td>9</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
After the values have been inserted, we set the diagram to a box plot by pressing the \( Y \) button.

We set the values to D1 and Freq.

We display the box plot by pressing the \( \text{Plot}\) button. By clicking directly on the elements of the box we obtain the statistical values:

- The minimum value at 54
- The first quartile at 65
- The median at 72
- The third quartile at 76
- The maximum value at 79
Bernoulli Schema: Binomial Distribution

HP Prime

**Model exercise:** A lottery drum contains 49 white balls and one gold ball. We win if we draw the gold ball.

1/ Calculate the probability that you draw a white ball, and the probability of winning.
2/ Prove that this is a Bernoulli trial and specify parameters.
3/ Perform 5 draws returning the lottery drum to its original state. Calculate the probability that you win 0 times, 1 time, 2 times, 3 times, 4 times and 5 times.
4/ Plot these probabilities using a bar chart.

**Step-by-step solution:**

1/ \( P(\text{draw a white ball}) = \frac{49}{50} = 0.98 \).

\( P(\text{draw a gold ball}) = 1 - 0.98 = 0.02 \).

2/ The experiment has two possible solutions: we either draw a white ball and lose, or draw the gold ball and win. Therefore this is the Bernoulli scheme, where the parameter \( n \) = number of draws, and the parameter \( p \) = probability of winning = 0.02.

3/ The HP Prime calculator is equipped with the command button \( \text{binomial}(n,k,p) \) which calculates the probability of \( k \)-multiple wins using the Bernoulli scheme with parameters \((n, p)\).

In our case, \( n = 5 \) draws.

By using this command button we obtain the sought probability.

4/ By pressing the \( \text{I} \) button we start the „Statistics 1Var“ application.

**Screenshots:**

```
<table>
<thead>
<tr>
<th>49</th>
<th>50</th>
<th>.98</th>
</tr>
</thead>
<tbody>
<tr>
<td>BINOMIAL(5,0,.02)</td>
<td>.9039207968</td>
<td></td>
</tr>
<tr>
<td>BINOMIAL(5,1,.02)</td>
<td>.092236816</td>
<td></td>
</tr>
<tr>
<td>BINOMIAL(5,2,.02)</td>
<td>.003764768</td>
<td></td>
</tr>
<tr>
<td>BINOMIAL(5,3,.02)</td>
<td>.000076832</td>
<td></td>
</tr>
<tr>
<td>BINOMIAL(5,4,.02)</td>
<td>.00000784</td>
<td></td>
</tr>
<tr>
<td>BINOMIAL(5,5,.02)</td>
<td>.00000032</td>
<td></td>
</tr>
</tbody>
</table>
```

By pressing the \( \text{I} \) button we start the „Statistics 1Var“ application.
In column D1, we insert the 6 values for the pre-calculated probabilities.

<table>
<thead>
<tr>
<th>D1</th>
<th>D2</th>
<th>D3</th>
<th>D4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.903920796</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>.09236816</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>.003764768</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>.000076832</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>.000000784</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>.000000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.000000032</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Press the Y button to select a chart type.

Press the V button to select the automatic measuring scale.

We only see two columns. The heights of the other 4 columns are very close to 0 (very low probability).
The Study of Function

HP Prime

Model exercise: A complex study of the irrational function $f$ defined as:

$$f(x) = \sqrt{x^2 + 4x + 3}$$

1/ Determine the intervals of monotonicity of the function.
2/ Find the infinite branches.
3/ Find the asymptotes.

Step-by-step solution:
1/ We define the function, from the $K$ window, by typing:

```
SAkRd
AwS
.Sjdj
+tsd+zE
```

We can differentiate the $f$ function by using the $f'$ notation in the copy:

```
Shift
Alpha
Log
Shift
> x \theta n
Alpha
6
Shift
```

The HP Prime calculator displays the derived function.

Note: The first two factors are equal to $x + 2$.
Because the denominator is a square root (always a positive number), the sign of the derivative is the sign $x + 2$.
Attention must be paid to the prohibited interval <-3; -1> in which the $f$ function is not defined.

Screenshots:

```
f:= (x) \rightarrow \sqrt{x^2 + 4x + 3}
(\rightarrow \sqrt{x^2 + 4x + 3})
f(x) = \frac{x^2 + 4x + 3}{2}\sqrt{2x+4}\sqrt{x^2 + 4x + 3}
```

```
f:= (x) \rightarrow \sqrt{x^2 + 4x + 3}
(\rightarrow \sqrt{x^2 + 4x + 3})
f(x) = \frac{x^2 + 4x + 3}{2}\sqrt{2x+4}\sqrt{x^2 + 4x + 3}
```

```
Sto \rightarrow simplif
```

```
Sto \rightarrow simplif
```

84
We can carry out a test.
The HP Prime calculator allows the determination of the derivative sign. We look up the solve command button using the CAS button in the menu Solve > Solve.

By using Shift (5), we obtain signs „equals”, „is greater” or „is less”. The HP Prime calculator displays all solutions, therefore, the variations of the \( f \) function.
- \( f \) is decreasing at \((-\infty; -3)\)
- \( f \) is undefined at \( -3; -1 > \)
- \( f \) is increasing at \((-1; +\infty)\)

The sign can be displayed using a graphical representation of the \( f \) function.
We run the „Function” application, we enter the expression of the function using \( Y \) beside F1, and by using \( P \) we display the chart.

2/ The graphical representation shows two infinite branches.

We find the limit of \( f(x)/x \) \( v +\infty \) and \( v -\infty \).
To find the limit symbol, press \( F \) in \( K \).
The \( \infty \) symbol can be obtained using Shift (8).

This will give us two final limits: 1 and -1. The branches are not parabolic, but controlled by linear oblique asymptotes with a slope of 1 in \( +\infty \) and -1 in \( -\infty \).
We specify the default coordinate y for equations of the asymptotes.
For this purpose, we calculate limit difference \( f \) with \( x \) in \( +\infty \) and subsequently limit difference \( ff \) for \( x \) with \(-x\) in \(-\infty\).
We get 2 and -2 as the default coordinates \( y \).

Therefore:
The function has an oblique asymptote with equation \( y = x + 2 \) in \( +\infty \), and oblique asymptote with equation \( y = -x - 2 \) in \(-\infty\).
Oblique asymptotes can be constructed by entering these two equations beside F2 and F3 symbolic display in the „Function“ application.

The display confirms our findings.
The Lucas-Lehmer primality test for Mersenne numbers is as follows:
Let $M_p = 2^p - 1$ be the Mersenne number to test. We define the sequence as follows: $s_0 = 4$; and $s_i = s_{i-1}^2 - 2$

The Mersenne number $M_p$ is a prime number only if $s_{p-2} = 0$ ($M_p$ model).

Write an algorithm that tests (using this method) the primality of any Mersenne number.

**Step-by-step solution:**
We will let the algorithm compute the successive terms of the sequence, and we test the essential and necessary conditions to achieve the desired position.

```plaintext
EXPORT LUCASLEHMER()
BEGIN
LOCAL M,P;
INPUT(P,"Enter an odd prime number");
2^P−1 M;
2 P I;
4 U;
WHILE U≠0 AND I≤P DO
  I+1 I;
  U*U−2 U;
  irem(U,M) U;
  IF I==P THEN
    IF irem(U,M)==0 THEN
      PRINT("Mersenne number 2^"+P+"−1=“+M+" is a prime number.");
    ELSE
      PRINT("Mersenne number 2^"+P+"−1=“+M+" isn’t a prime number.");
    END;
  END;
END;
END;
END;
```

The determined prime number can be verified using the `isPrime` command button. If the number isn't a prime number, 0 is displayed; if the number is a prime number, 1 is displayed.

**Screenshots:**

- **HP Prime: LUCASLEHMER**
  - `EXPORT LUCAS_LEHMER()`
  - `BEGIN`
  - `LOCAL M,P;`
  - `INPUT(P,"Enter an odd prime number");`
  - `2^P−1 M;`
  - `2 P I;`
  - `4 U;`
  - `WHILE U≠0 AND I≤P DO`
  - `  I+1 I;`
  - `  U*U−2 U;`
  - `  irem(U,M) U;`
  - `  IF I==P THEN`
  - `    IF irem(U,M)==0 THEN`
  - `      PRINT("Mersenne number 2^"+P+"−1=“+M+" is a prime number.");`
  - `    ELSE`
  - `      PRINT("Mersenne number 2^"+P+"−1=“+M+" isn’t a prime number.");`
  - `    END;
  - `  END;
  - `END;
  - `END;

The Mersenne number $2^{11}−1=2047$ is not prime.
The Mersenne number $2^{19}−1=524287$ is first.
The Mersenne number $2^{23}−1=8388607$ is not prime.
Create the table below using an algorithm.
The first column consists of 1, and every other value in the table is obtained by adding the two nearest elements: one which is located on the row above and one to the left of that one.

<table>
<thead>
<tr>
<th>n= 0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

Step-by-step solution:
The user is asked to enter the size of the required triangle (the n value).
The use of a matrix provides an easy and interesting solution in order to create Pascal’s triangle. We create an nxn matrix and specify individual coefficients using the above addition method.

```plaintext
EXPORT PASCAL()
BEGIN
INPUT(N);
//We create an nxn matrix
MAKEMAT(0,N+1,N+1) M1;
FOR I FROM 1 TO N+1 DO
//We fill in the matrix so that we enter 1 in the first column and in the external diagonal
M1(I,1):=1;
M1(I,I):=1;
END;
FOR I FROM 3 TO N+1 DO
FOR J FROM 2 TO I−1 DO
M1(I,J):=M1(I−1,J−1)+M1(I−1,J);
END;
END;
//We display each line separately on the console display
PRINT;
FOR I FROM 1 TO N+1 DO
PRINT(M1(I));
END;
END;
```

Screenshots:
Now we are interested in the following equation:
\[ X^2 - 5X - 6 = 0. \]

Here we see the result for \( n = 6 \).

The number, which is located at the intersection of the \( n \) row and the \( p \) column represents the coefficient of the \( p \) position in its expanded form \((x+y)^n\) (Newton's binomial theorem).

This number is called a binomial coefficient and is marked as \( C(n, p) \). It is expressed by the following formula:

\[
C(n,p) = \frac{n!}{(n-p)! \times p!}
\]

The HP Prime calculator is equipped with the \textit{COMB} command button which is used for direct calculation of these binomial coefficients.

And finally, one useful tip: For fast calculation of a Pascal's triangle row, we can ingeniously use Newton's binomial theorem: we will raise to a power the position of line 11 (on 4 rows) and 101 (on 4 rows) and 1001 (on 4 lines), etc.
Sequences and the Sigma Symbol

HP Prime

**Model exercise:** The \((u_n)\) sequence is given for a positive integer that is not zero:

\[ u_n = \frac{1}{n} \sum_{k=1}^{n} k(k - 1) \]

1/ Calculate the first three terms of the sequence.
2/ Using a spreadsheet, display the first 30 terms of the sequence.
3/ The sequence \((v_n)\) is given using the formula \(v_n = u_{n+1} - u_n\). Display the sequence \((v_n)\) using a chart.

**Step-by-step solution:**
1/ On the HP Prime calculator, we insert the sigma character by pressing the \(F\) button.

Now we can calculate the first three terms of the sequence.

---

**Screenshots:**

- **Stats - 1Var**

- **Stats - 1Var**

- **Stats - 1Var**

- **Stats - 1Var**
2/ Using the I button, run the „Sequence“ application.

Insert the sequence expression \((u_n)\).

Press the M button to get all the values for the following consecutive terms of the sequence \((u_n)\).
3/ Press the \(Y\) button to define the sequence \((u_n)\). Press the \(Eval\) tab for evaluation and activation of the sequence.

**Useful tip:**
By pressing buttons > and <, it is possible to move along the curve from term to term. Press + or - to zoom in or out.
Tangent to the Curve

**HP Prime**

**Model exercise:** Determine the equation of the tangent to the curve representing the function $y = -2x^5 + \tan x$ at point 7. Construct the tangent.

**Step-by-step solution:**

Use the $\underline{\text{I}}$ button to access the application.

Beside $F_1(X) =$ enter the algebraic expression of the function using successive presses of the following buttons:

- $6$ $\rightarrow$ $1$ $\rightarrow$ $9$ $\rightarrow$ $d$ $\rightarrow$ $E$

Press the $\underline{\text{P}}$ button for a graphical representation of the function. Press $> >$ and select “Tangent”. Use $<$ and $>$ for the movement along the curve. The tangent is displayed at each point in dotted lines. Press “Go to” to go to $x = 7$ and confirm by pressing the $\underline{\text{E}}$ button.

For a tangent equation we use the formula $y = f'(7)(x - 7) + f(7)$.

**Screenshots:**

- **Application Library:**
  - Function
  - Advanced Graphing
  - Geometry
  - Spreadsheet
  - Statistics 1Var
  - Statistics 2Var
  - Inference
  - DataStreamer
  - Solve
  - Linear Solver
  - Quadratic Explorer
  - Trig Explorer

- **Function Symbolic View**

- **Graphical representation:**
  - Fcn
  - 1 Root
  - 2 Slope
  - 3 Signed area...
  - 4 Extremum
  - 5 Tangent

Press the $\underline{\text{Plot}}$ button for a graphical representation of the function and select “Tangent”.

**Model exercise:** Determine the equation of the tangent to the curve representing the function $y = -2x^5 + \tan x$ at point 7. Construct the tangent.

**Step-by-step solution:**

Use the $\underline{\text{I}}$ button to access the application.

Beside $F_1(X) =$ enter the algebraic expression of the function using successive presses of the following buttons:

- $6$ $\rightarrow$ $1$ $\rightarrow$ $9$ $\rightarrow$ $d$ $\rightarrow$ $E$

Press the $\underline{\text{P}}$ button for a graphical representation of the function. Press $> >$ and select “Tangent”. Use $<$ and $>$ for the movement along the curve. The tangent is displayed at each point in dotted lines. Press “Go to” to go to $x = 7$ and confirm by pressing the $\underline{\text{E}}$ button.

For a tangent equation we use the formula $y = f'(7)(x - 7) + f(7)$.
The derivative for a single point can be calculated using the \textit{SLOPE} command button:

\[ F_1(x) = -2x^5 + \tan(x) \]
\[ F_2(x) = \text{SLOPE}(F_1, 7) + (x-7) + F_1(7) \]
**Integral**

**HP Prime**

**Level:** The third (graduation) year of the science branch of French Lyceums

**Objectives:** verifying the hypothesis, writing and the use of an algorithm

**Keywords:** Algorithm, integral, surface area.

**Task:** We have function $f$ defined on $\mathbb{R}$ as $f(x) = (x + 2)e^{-x}$.

We mark the curve showing the function $f$ in an orthogonal coordinate as $D$.

1. Find the intervals of monotonicity of the $f$ function on $\mathbb{R}$.

2. We mark the domain between the axis of coordinates $x$, the curve $C$ and straight lines $x = 0$ and $x = 1$ as $D$.

We first calculate an approximate surface area of the $D$ domain so that we calculate a sum of areas of rectangles.

We divide the interval $[0; 1]$ into four intervals of the same length.

```
Create an algorithm to obtain an approximate value of the area of the $D$ domain by adding up the areas of all four previous rectangles.
```

3. Calculate the surface area rounded to $10^{-3}$ which you obtain by using this algorithm.

4. Now we divide the interval $[0, 1]$ into $N$ equal intervals.

Change the algorithm so the output displays the sum of areas of $N$ identical rectangles.
Step-by-step solution:
1/ By reviewing the sign of the derivative of the function we determine the intervals of monotonicity: a non-decreasing on (-∞, -1) and non-increasing on (-1, +∞). In our examined interval [0, 1], the function is decreasing.

On the HP Prime calculator, the expression for the derivative of the function may be obtained directly by the \( \mathbf{K} \) button. The syntax for the derivative is available using the \( \mathbf{F} \) button.

We differentiate \( F4 \) (the expression entered using the \( Y \) button: see the first screenshot).

When inserting the calculation of the derivative, we use the formal variable \( x \) in lowercase.

The \texttt{solve} command button provides zeros of the derivative: it is zero at -1 (which can be determined by dividing the \texttt{exp(-x)}).
A graphical representation of the function can be obtained by pressing the `Plot` button.

2/ In the programme, we will create the FOR loop to merge the areas of rectangles. The rectangle area is obtained by multiplying its width by \(1/4\) (1 divided by the number of rectangles) and its length: \(f(0)\) for the first rectangle, \(f((k-1)/4)\) for the k-th rectangle.

A and B denote the limits of the examined interval.

We can expand the programme and construct rectangles using the command button `RECT_P`.

3/ We carry out the algorithm that displays the approximate value of the surface area under the curve, rounded first up and then down. For \(n = 50\) we obtain the values shown in the opposite picture (see question 4).

Now we can use the module/processor of the HP Prime calculator for a formal calculation of integrals. Press the `K` button and using the `F` button find the integral symbol.
Insert the integral using successive presses of the buttons below:

The HP Prime calculator displays the exact value of the integral. By pressing the button, we obtain a rounded decimal value that lies between the two limits, which we calculated using the algorithm.

4/ Now it is only necessary to add INPUT(N); at the beginning of the programme the user is asked the number of rectangles in the division; and replace 4 rectangles with N rectangles.
Calculating Area between Two Curves

HP Prime

From a practical test of the natural science branch, June 2008.

Level: The third (graduation) year of the science branch of French Lyceums

Objectives: Functions, geometric interpretation of an integral of the difference of two functions.

Keywords: Functions, integrals, surface area.

Task: Determine the surface area between the curve representing the function $f(x) = \ln(x)$, and the curve representing the function $g(x) = (\ln(x))^2$ for $x$ in the interval 1 to $e$.

Step-by-step solution:
Solving the task using a chart on the HP Prime calculator:
First open the “Function” application using the I button.

Enter the two functions $f$ and $g$ beside F1(X) = and F2(X) =.

Screenshots:

Application Library

Function
Advanced Graphing
Geometry
Spreadsheet
Statistics 1Var
Statistics 2Var
Inference
DataStreamer
Solve
Linear Solver
Quadratic Explorer
Trig Explorer

Function Symbolic View

$\sqrt{ }$
F1(X) = LN(X)
F2(X) = LN(X)^2
$
\sqrt{ }$
F3(X) =
F4(X) =
F5(X) =
F6(X) =
F7(X) =
Enter function

Edit  \sqrt{}  \times  \boxed{}  Show  Eval
Press the button to display the curves representing the functions (for greater clarity, the curves are in different colours).

Using the combination, it is possible to set limits and the chart scale. Since both functions are defined for $x > 0$, we set the minimum $x$ coordinate to 0. The task requires a calculation of the surface area for $x = 1$ to $e$, and therefore we set the maximum $x$ coordinate to 3. We set the minimum of $y$-axis to -1 and the maximum of $y$-axis to 2.

By pressing the button, we will again display the curves and the surface area, which divides them by the desired interval. This includes the examined interval since the two curves intersect at $x = 1$ and $x = e$. We can verify this because the HP Prime calculator displays coordinates of intersections of both curves.
Press Menu to activate tools for the analysis and select Fcn > Intersection.
Select „Intersection“...

Then F2(X).

We obtain the coordinates of the first intersection: (1; 0).
The HP Prime calculator allows colour coding and enables calculating the surface area between the two curves for the desired interval. For this purpose press \( \text{Menu} \rightarrow \text{Fn} \) and select „Signed area...“.

Place the cursor at \( x = 1 \) by pressing \( \text{Go To} \) and enter \( X \) as the value for \( x \).

Confirm by \( \text{OK} \) and select F2(X). Then move the cursor to \( x = e \) by pressing \( \text{Go To} \) and enter \( e \) as the value for \( x \). To enter the \( e \) symbol, press this button sequence: \( \text{Shift} \ \text{COS} \) and cancel the exponent.
The area between the two curves will be coloured in.
Confrim by pressing OK.
The calculator displays the value for the area at the bottom of the screen.
This value may be verified by calculating the integral of the difference \( f - g \) between the limits 1 and \( e \) which geometrically corresponds to the surface area between the two curves in the interval \([1; e]\).
The relative position of the two curves can be obtained using the sign table below:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 0 )</th>
<th>( 1 )</th>
<th>( e )</th>
<th>( +\infty )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \ln(x) )</td>
<td>( -)</td>
<td>( 0 )</td>
<td>( + )</td>
<td>( + )</td>
</tr>
<tr>
<td>( 1 - \ln(x) )</td>
<td>( + )</td>
<td>( + )</td>
<td>( 0 )</td>
<td>( - )</td>
</tr>
<tr>
<td>( \ln(x) \times (1 - \ln(x)) )</td>
<td>( - )</td>
<td>( 0 )</td>
<td>( + )</td>
<td>( 0 )</td>
</tr>
</tbody>
</table>

The curve of function \( f \) is, in the examined interval, above the curve of function \( g \).

In order to calculate the integral on the HP Prime calculator, press the \( K \) button to get into the window of the formal calculation.
Find the integral character using the \( F \) button.

Then enter the difference of the integrals and fill in limits and terms:
Using the \( W \) button, it is possible to directly display the approximate decimal value of the result.
We reached the same result that is displayed in the window displaying charts.

The integral can be calculated using the integration parts of the integral \( \ln(x) \) and using the auxiliary function \( G(x) = x(\ln(x)^2 - 2\ln(x) + 2) \).
Step-by-step solution:
The HP Prime calculator can store complex numbers in variables Z0 and Z9.
Writing of a complex number is performed using buttons Shift [ y ].

1/ Now we can perform direct calculations required for Z1 and Z2.

2/ In the calculation window, press [ y ] to access the command buttons for complex numbers in the list.
The argument is calculated using the ARG command button. The modulus is calculated by using the ABS command button.

Useful tip: The IM( command button allows to display the imaginary part of a complex number and the REC command button displays its real part.
1/ Determine the size of a directed angle using an algorithm.
2/ Test the algorithm using $123\pi/4$.

**Step-by-step solution:**
The size of the angle is within the interval $(-\pi; \pi)$. Successive multiples of $2\pi$ will be added to or subtracted from the size of the given angle until the interval is reached. In order to avoid an inaccurate calculation at the output, the best solution is to consider $X$ as the fraction $P/Q$ of the $\pi$ factor and to process $P$ and $Q$.

**Begin**
Input $P$
Input $Q$

**Processing:**
If $P/Q \geq 0$ Then
While $\text{ABS}(P/Q) > 1$
$P$ will have a value of $P+2Q$
End While
Else
While $\text{ABS}(P/Q) > 1$
$P$ will have a value of $P-2Q$
End While
End If

**Output**
Print $P/Q.\pi$

The $P+2Q$ we acquired from the $P/Q.\pi + 2\pi = (P+2Q).\pi/Q$
This is also true for $P-2Q$.
In order to avoid an inaccurate calculation at the output, we display the fraction / and the $\pi$ as character strings.
The programme displays the exact size $123\pi/4$.

**Screenshots:**
We rewrite this algorithm to the HP Prime:

```
BEGIN
INPUT(P);
INPUT(Q);
IF P/Q>=0 THEN
WHILE ABS(P/Q)>1 DO
P:=P+2Q;
END;
ELSE
WHILE ABS(P/Q)>1 DO
P:=P-2Q;
END;
END;
PRINT(P+"/"+Q+"\pi");
END;
```

```
END;
```

The programme displays the exact size $123\pi/4$. 

---

**Date of issue:** 06.2014
The Square Root Approximation

HP Prime

Objectives: Calculate an approximate value of the square root using a recurrent sequence, write an algorithm.

Keywords: Sequence, recurrence, algorithm, square root.

Task: The following algorithm is given for the square root approximation of the X number:
- We choose the default number Y.
- We calculate a half-sum of Y and X/Y.
- We assign this result to Y and start again.
Run the algorithm.

Assign the algorithm to a sequence that has the tendency $\sqrt{X}$.

Step-by-step solution:
We can start by writing the algorithm in the generic form:

Variables:
- X (for which we want the square root approximation)
- Y starting number
- N (number of iterations)
- I (counter)

Inputs:
- Request X
- Request Y
- Request N (the number of iterations to calculate)

Processing:
- For I in the interval 1 to N do
- Assign $(Y + X/Y)/2$ to Y
- End For

Output:
- Print Y

For $X = 2$, $Y = 1$ and $N = 100$ we obtain:
I.e. the correct approximation $\sqrt{2}$. 

The algorithm will only calculate terms of the following sequence:
$U_{n+1} = (U_n + X/U_n)/2 \quad U_0 = Y$

Screenshots:

```
EXPORT RACINE()
BEGIN
    LOCAL I;
    INPUT(X);
    INPUT(Y);
    INPUT(N);
    FOR I FROM 1 TO N DO
        Y := (Y + X/Y)/2;
    END;
    PRINT(Y);
    END;

1.41421356238
```
The sequence may be calculated on the HP Prime calculator by running the “Sequence” using the ▼ button.

Insert the first term Y (in this example Y = 1) to U(1) and then insert the expression for the recurrent sequence to U1(N):

Set the representation to the network diagram mode by pressing the S and P buttons. Set also the extreme values.
Press the button for a graphical representation of the function.

We can zoom in on the part that interests us. Press >. Select a section of the window (which will represent the top left corner of the zoomed rectangle) and then move to the next point (which will represent the bottom right corner of the rectangle).

**Useful tip:** Press + or w to zoom in or out.

The sequence quickly converges to $\sqrt{2}$. This can be proven by a query $U_{n+1} = f(U_n)$.

In this example $f(x) = 1/2(x+2/x)$. It's sufficient to solve the equation $f(l) = l$.

We obtain $2l = l + 2/l$ or $l = 2/l$, therefore $l^2 = 2$. Therefore $l = \sqrt{2}$, because $U_0$ is positive and therefore all terms are positive. Note: We consider the first term $U_0 = Y$ as non-zero, because otherwise we would obtain a constant zero sequence.
Chinese Remainder Theorem

HP Prime

In this task, we want to determine all relative integers $N$ of the type

\[
\begin{align*}
N &= 5 \pmod{13} \\
N &= 1 \pmod{17}
\end{align*}
\]

**Step-by-step solution:**
The HP Prime calculator is equipped with a command button that allows instant solving of this task.

It is accessible using the $\checkmark$ button and is called $\text{ichinrem}$.

We enter the following form:

\[
\begin{align*}
\text{ichinrem}([5, 13], [1, 17]) &= [-203, 221]
\end{align*}
\]

We obtain the solution: all integers congruent to $-203$ modulo $221$, i.e. congruent to $18$ modulo $221$. 

Screenshots:
The following task will allow to prove the findings:

a) Prove that 239 is the solution to this system.

b) Let N be the relative integer as a solution to this system. Prove that N can be written in the following form
\[ N = 1 + 17x = 5 + 13y, \]
where \( x \) and \( y \) are two relative integers to verify the relationship \( 17x - 13y = 4 \).

c) Solve the equation \( 17x - 13y = 4 \), where \( x \) and \( y \) are relative integers.

d) Conclude that there is a relative integer \( k \) of type \( N = 18 + 221k \).

e) Prove equality between \( N \equiv 18 \pmod{221} \) and
\[
\begin{cases}
N \equiv 5 \pmod{13} \\
N \equiv 1 \pmod{17}
\end{cases}
\]
In a sample of 10,000 individuals of a given population, 7.5% of those are treated for elevated cholesterol. Calculate the interval in which we have 95% „certainty” that we can find the exact number of people from the 10,000 which need to be treated.

**Step-by-step solution:**
The HP Prime calculator has the tools necessary to directly obtain the confidence interval sought. Run the „Inference“ application using the APPS button.

Press the \( Y \) button to adjust the methodology to „Confidence Interval“ and the type of \( \text{Int Z: } 1 \pi \)
Press the button to enter the initial data of the task. \( n \) is the number of people. 
\( x \) is the number of people with high cholesterol: 
\[ 0.075 \times 10\,000 = 750. \]
\( C \) is the confidence level: 0.95.

Press the button to display the interval sought:
\[ 0.0698 \times 10\,000 = 698 \text{ persons} \]
\[ 0.0802 \times 10\,000 = 802 \text{ persons} \]
Probability: The Normal (Gaussian) Probability Distribution

HP Prime

Step-by-step solution:
1/ On the HP Prime calculator, it is possible to calculate probabilities using the normal (Gaussian) distribution. For this purpose it is necessary to use the `normal_cdf`, command button followed by both the parameters (the mean value $m = 22$ and the standard deviation $σ = 4$) for a normal distribution of parameters $N(m, σ²) = N(22.4²)$, and the upper limit of $19°C$.
To calculate $P(T ≤ 19)$ we type:
\[
\text{normal_cdf}(22,4,19)
\]
The probability that the temperature in July will be lower than $19°C$ is $≈0.23$.

2/ \( P(T ≥ 27) = 1 − P(T ≤ 27) \)
Therefore, we type:
\[
1 − \text{normal_cdf}(22,4,27)
\]
The probability that the temperature in July will be higher than $27°C$ is $≈0.11$.

3/ \( P(24 ≤ T ≤ 30) = P(T ≤ 30) − P(T ≤ 24) \).
Therefore, we type:
\[
\text{normal_cdf}(22,4,30) − \text{normal_cdf}(22,4,24)
\]

Model exercise: The temperature $T$ in July evolves according to a normal distribution with an average (mean) value of $22°C$ and a normal standard deviation of $4°C$.
1/ Calculate the probability that the temperature will be lower than $19°C$.
2/ Calculate the probability that the temperature will be higher than $27°C$.
3/ Calculate the probability that the temperature will be within the interval of $24°C$ to $30°C$.
4/ Find the temperature $t$, for which $P(T ≤ t) = 0.8$.
5/ Plot the probability density $f$ for $T$.
6/ What is the $P(30 ≤ T ≤ 35)$ on the chart?

Screenshots:

- $\text{NORMALD}_\text{CDF}(22,4,19)$
  - $0.226627352377$

- $1−\text{NORMALD}_\text{CDF}(22,4,27)$
  - $0.105649773667$

- $\text{NORMALD}_\text{CDF}(22,4,30)−\text{NORMALD}_\text{CDF}(22,4,24)$
  - $0.285787406778$
The probability that the temperature in July will be within the interval of 24°C to 30°C is ≈0.29.

4/ We use the reverse command button `normal_icdf`.
Therefore, we type:

\[ \text{normal_icdf}(22,4,0.8) \]

The probability \( P(T \leq t) = 0.8 \) for \( t = 25.4°C \).

5/ The probability density \( f \) of the \( T \) value can be calculated using the command button `normald(f(x)=normald(22,4,x)`

In the „Function” application, it is possible to enter

\[ F1(X) = \text{normald}(22,4,X) \]

The window display is set using the \( \text{S} \) and \( \text{P} \) buttons. We can configure the following settings to display the resulting diagram using the \( \text{P} \) button.

6/ The probability \( P(30 \leq T \leq 35) \) that the temperature will be 30°C to 35°C, is shown graphically by the area under the curve between the coordinates \( x = 30 \) and \( x = 35 \) (the surface area defined by the line equation \( x = 30, x = 35 \) and \( C \)).
This situation can be displayed on the HP Prime calculator by pressing the following in the graphical display window: First we press [Menu] and then [Fn]. we select „Signed area”, we press [Go To] to enter \( x = 30 \), then [OK], and again [Go To] to enter \( x = 35 \) and at the end [OK].

The window can be set so as to better see the hatched zone (Shift Plot2 Set).

Function Plot Setup

| X Rng: 29 | 36 |
| Y Rng: 0 | 0.02 |
| X Tick: 1 |
| Y Tick: 1 |

Enter maximum vertical value
Random Walk

HP Prime

Objectives: Verifying the hypothesis, writing and the use of an algorithm.

Keywords: Algorithm, iteration, while loop.

Task: A pawn is placed on the starting square of the board:

| pawn | pawn | pawn | pawn | pawn | pawn | pawn | pawn | pawn | pawn |

A coin-toss determines the movement of the pawn: HEADS = the pawn will move to the right; TAILS = The pawn will move to the left. Each toss will get assigned a real number +1, if it is a HEAD; and -1, if it is a TAIL. The route consists of a sequence of n moves. The random variable Sn is the sum of the numbers 1 or -1, corresponding to n tosses along the route.

We are interested in the Dn event: „After n moves on the route, the pawn moved back to the starting square.“ The following algorithm allows the simulation of the route as the resultant of n moves; the user can choose the n value.

Variables:
N,S,A: real numbers

Processing:
Input N
S will have a value of 0
For I variations in the interval 1 to N
A will have a value of a random integer 0 or 1
If A=1
Thus S will have a value of S+1
Otherwise, S will have a value of S-1
End If
End For

Output:
Print S
End

1/ Use this algorithm on the calculator to perform multiple simulations where the pawn performs 1 or 2 moves.

2/ Adjust the above algorithm so you can perform a simulation of the pawn’s several routes and calculate the frequency of the Dn event.
Step-by-step solution:

1/ We adjust the algorithm on the HP Prime calculator.

Input 1 or 2 for N to perform the algorithm. The programme will display a random variable Sn, which also corresponds to the position of the pawn (0 for the starting square, +1 for 1 square after the starting square, -2 for 2 squares before the starting square, etc.).

2/ The previous algorithm needs to be run several times to perform the simulation of several routes. We store each route in a list, or view in succession individual values S.

We run the algorithm below:

Variables:
X, I: integers

Processing:
Input X (number of simulated routes)
Let L be an empty list
For I variations in the interval 1 to X
Run the MARCHE (WALK) programme
Add the S as an element of the L list
End For

Output:
Print L
End

Screenshots:

```plaintext
BEGIN
LOCAL A, S;
INPUT(N);
0*S;
FOR I FROM 1 TO N DO
ROUND(RANDOM(0,1),0)*A;
IF A==1 THEN
S+1*S;
ELSE
S-1*S;
END;
END;
PRINT(S);
END
```
The programme will display a list of squares to which the pawn moved (in our example, the result of 8 simulations and 2 moves).

To determine the frequency of the $D_n$ events, we divide the number 0 in the list by the number of elements in the list. We will increment the counter to calculate the 0's.

\{0,2,-2,-2,2,0,2\}
\{0,0,0,2,0,0,0\}
\{0,-2,2,-2,0,2,2,-2\}
\{-2,0,0,2,2,2,-2\}

```c
EXPORT MARCHE()
BEGIN
LOCAL A,S,I,J,C;
INPUT(N); INPUT(X):
{}\rightarrow L1;
0\rightarrow C;
FOR I FROM 1 TO X DO
0\rightarrow S:
FOR J FROM 1 TO N DO
ROUND(RANDOM(0,1),0)\rightarrow A;
IF A==1 THEN
S+1\rightarrow S;
ELSE
S-1\rightarrow S;
END:
END;
CONCAT(L1,{S})\rightarrow L1:
IF S==0 THEN
C+1\rightarrow C;
END:
END;
PRINT(C/X);
END;
```

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Graduation Task Solution

HP Prime

Sample graduation task of the science branch of French Lyceums, 2013 (Metropolitan France - June - Task 2).

In the following graph in a coordinate system with orthonormal basis \((0; \vec{i}, \vec{j})\) we have a marked curve \(y\) of function \(f\) that is defined and differentiable in the interval \((0, +\infty)\).

Are given by the following information:

- points \(A, B, C\) have coordinates \([1, 0], [1, 2], [0,2]\)
- curve \(y\) intersects point \(B\) and line \(BC\) touches the curve \(y\) at point \(B\)

\[ f(x) = \frac{a + b \ln x}{x} \]

1. a. Using the graph to identify the values of \(f(1)\) and \(f'(1)\).
   b. Show that for all real positive \(x\) is:
   c. Calculate values of \(a, b\).

2. a. Prove that for all real \(x\) from the interval \((0, +\infty)\) have \(f'(x)\) the same sign as \(\ln x\).
   b. Specify the limit of a function \(f\) at 0 and at \(+\infty\). We will be able to specify that for all real \(x\) is positive:
   c. Investigate the intervals of monotonicity of \(f\)

3. a. Prove that the equation \(f(x) = 1\) have only one solution \(\alpha\) in the interval \((0, 1)\).
   b. In an analogous way, prove that there is only one real \(\beta\) in the interval \((1, +\infty)\), such that \(f(\beta) = 1\).
   Determine an integer of \(n\) so that the \(n\) is true: \(\beta < n + 1\).
4. The following algorithm is given:

**Variables:** $a, b, m$ are real numbers.

**Inputs:** Assign a value 0 to a variable
Assign a value 1 to b variable

**Processing:** condition (until the condition ...) $b - a > 0.1$
Assign the value of $m = \frac{1}{2} (a + b)$
If $f(m) < 1$, then assign a value to the variable $m$
If not, assign a value to a variable $b$ and end conditions.

**Output:** List $a$.
List $b$.

a. Allow the run of this algorithm and continuously replenish the following table:

<table>
<thead>
<tr>
<th></th>
<th>step 1</th>
<th>step 2</th>
<th>step 3</th>
<th>step 4</th>
<th>step 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b - a$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. What are the values that we obtained from this algorithm?

c. Change the algorithm so that it shows both border frame with amplitude $\beta = 10^{-1}$

5. The purpose is now to prove that the curve $OABC$ divides the rectangle into two equal areas.

a. To prove use the integral: $\int_{1/e}^{1} f(x) \, dx = 1$.

b. Note that the expression $f(x)$ can be written as $\frac{2}{x} + 2 \frac{1}{x} \ln x$, and complete the example.
Step-by-step solution:

1/ a/ The first question implies a "reading" of the diagram: $f(1)$ is the representation of 1 using a function. It corresponds to the Y coordinate of point B: 2.
Therefore $f(1) = 2$.
f'(1) corresponds to the slope of the tangent to the curve representing $f$ in 1.
The tangent is horizontal, therefore, $f'(1) = 0$.

b/ We can find which derivative the HP Prime calculator will display by pressing the K button.
We find the derivative using the F button.
We insert the parameters a and b in lower-case letters

The result won’t be displayed in the form of a single quotient.
To convert an expression to a common denominator, we press “simplify” in the window.
We find the expression of the task.
To determine the detailed calculation of the derivative, we use the formula $(u/v)' = (u'v-uv')/v^2$.

c/ We apply both 2 equations determined in 1/ $f(1) = 2$ and $f'(1) = 0$. We obtain a system of two equations with two unknowns a and b. We can invoke and then use the solve command button of the HP Prime calculator to perform the solution.
We also use the symbol |, which means that the expression is evaluated for a given value of the selected argument. Press the F button and the below symbol for the evaluation of expressions in \( x = 1 \).

| \( \sqrt{a} \) | \( \sqrt{b} \) | \( 1 \) | \( a \) | \( b \) | \( a \times b \) | \( |a-b| \) |
|---|---|---|---|---|---|---|
| \( \sqrt{a} \) | \( \sqrt{b} \) | \( 1 \) | \( a \) | \( b \) | \( a \times b \) | \( |a-b| \) |

We find that \( a = 2 \) and \( b - a = 0 \), therefore \( a = b = 2 \).

2/ a/ Therefore, the derivative \( f \) has the expression (if \( a \) and \( b \) is replaced by 2): \(-\ln(x^2)/x^2\).

3/ Because \( x^2 \) is still a positive number, the derivative, therefore, has the same sign as \(-\ln(x^2) = -2\ln(x)\), i.e. the same sign as \(-\ln(x)\).

b/ We press the F button again to calculate limits. We insert:

For zero the HP prime calculator indicates + or - infinity. Since \( f \) is defined solely and only for positive numbers, to specify to the right of 0, we write \( 0^+ \) (to specify to the left of 0, we write \( 0^- \)). Therefore, we find \(-\infty\) as the limit in 0.

We use the other possible expression of the function \( f \) and the limit operation to verify this:

The limit \( f \) at infinity is 0. The \( \infty \) symbol can be obtained using the buttons.
c/ Now we can graphically display the $f$ function and determine its variations.

Access to the “Function” application can be obtained by pressing the $I$ button. We insert an algebraic expression of the function using the $Y$ button.

The diagram can be displayed by pressing the $P$ button. The scale is set automatically using the $V$ button. Then we can slightly reduce the extremes of $y$ coordinates using the $SP$ buttons.

We take $y_{\text{min}} = -2$ and $y_{\text{max}} = 3$. 
The \( f \) function is increasing on \((0; 1)\) and decreasing on \((1; +\infty)\). Based on the review of the derivative sign, we can build the following variation table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>(+\infty)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-\ln x)</td>
<td>+</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>( f(x) )</td>
<td>(-\infty)</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

We find that the graphical representation given in the task corresponds to the graphic expression of \( f \).

3/ a/ Since the function \( f \) is strictly increasing continuously on \((0; 1)\) and because 1 is located in the interval between the limits \( f \) in 0 and \( f(1) \), Bolzano's theorem is the only solution for \( f(x) = 1 \).

b/ We display the table of values of the function \( f \) using the button. \( f \) has a value of 1 in the interval 5 and 6.

**Useful tip:**

We press \( \text{View} \) and select 2: „Split Screen: Plot Table“ to display simultaneously (in a split window) the window with a chart and the table with values.
On the HP Prime calculator, we programme the algorithm in the programme editor by pressing the \texttt{Shift} \texttt{×} buttons.

\begin{verbatim}
EXPORT BACS()
BEGIN
LOCAL A,B,M;
0 A;
1 B;
WHILE B–A>0.1 DO
(A+B)/2 M;
IF F1(M)<1 THEN
M A;
ELSE
M B;
END;
END;
PRINT(A);
PRINT(B);
END;
\end{verbatim}

If we want to display the individual required stages in the table, it is necessary to change the algorithm so that the PRINT tags are placed in the while loop and we add imaging \texttt{b-a} and \texttt{m}. We can also display the stage number so that we create a counter:

\begin{verbatim}
EXPORT BACS()
BEGIN
LOCAL A,B,M,C;
0 A;
1 B;
1 C;
PRINT("Stage1");
PRINT(A);
PRINT(B);
PRINT(B-A);
C+1 C;
WHILE B–A>0.1 DO
(A+B)/2 M;
IF F1(M)<1 THEN
M A;
ELSE
M B;
END;
PRINT(C);
END;
\end{verbatim}
The programme then displays all the stages. Now it only remains to add the table:

<table>
<thead>
<tr>
<th></th>
<th>step 1</th>
<th>step 2</th>
<th>step 3</th>
<th>step 4</th>
<th>step 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0</td>
<td>0</td>
<td>0.25</td>
<td>0.375</td>
<td>0.4375</td>
</tr>
<tr>
<td>b</td>
<td>1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>b - a</td>
<td>1</td>
<td>0.3</td>
<td>0.25</td>
<td>0.125</td>
<td>0.0625</td>
</tr>
<tr>
<td>m</td>
<td>0.5</td>
<td>0.25</td>
<td>0.375</td>
<td>0.4375</td>
<td></td>
</tr>
</tbody>
</table>

b/ The proposed algorithm will display, in parallel, both limits for \( \alpha \) with an accuracy of 0.1.

c/ Again we come out of the initial algorithm and replace only the starting values \( A \) and \( B \) 5 and 6 instead of 0 and 1. In the test we also substitute if \( F_1(M) < 1 \) for \( F_1(M) > 1 \), because the \( f \) function is decreasing on \( (1; +\infty) \):

```
EXPORT BACS()
BEGIN
LOCAL A,B,M;
5 A;
6 B;
WHILE B–A>0.1 DO
(A+B)/2 M;
IF F1(M)>1 THEN
  M A;
ELSE
  M B;
END;
END;
PRINT(A);
PRINT(B);
END;
```

5/ a/ We start by calculating the area of the OABC rectangle whose length is 2 and width 1. Its area therefore consists of 2 area units.
To find the lower limit of the integral that will calculate the area under the curve of the function, it is necessary to solve \( f(x) = 0 \).
We can use the solve command button in the window of formal calculations (the K button).
The HP Prime finds a solution: x = 1/e.
The solution can be found very easily „manually“.

In the interval (1/e; 1), the f function is positive and continuous, and therefore the area defined by the curve of the f function, the axis of the coordinates x and line equations x = 1/e and x = 1 is given by the integral:

\[ \int_{1/e}^{1} f(x) \, dx \]

It is necessary to prove that it is equal to half of 2 (the rectangle area), i.e. 1.

To calculate the integral character select the „integral“ using the F button and enter:

The HP Prime calculator displays the correct result 1.

We can calculate the integral with change of variables, so that we use the expression offered for f and enter u = ln. We find the expression in the form f = 2u’ + 2u’u original F = 2u + u^2.

I.e. F(x) = 2ln(x) + ln(x)^2.

And F(1) – F(1/e) = 0 – 2ln(1/e) – ln(1/e)^2 = 2 – 1 = 1.